VOLUME I

PERFORMANCE FLIGHT TESTING

CHAPTER 5

PITOT-STATICS AND THE STANDARD ATMOSPHERE

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This textbook, *Pitot Statics and the Standard Atmosphere*, represents a consolidation of available data on pitot-static theory and current air data system calibration flight test techniques. It was written to provide USAF Test Pilot School students with a single, concise reference for the subject.

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5.1 INTRODUCTION

The propulsive and aerodynamic forces acting upon an airborne vehicle are functions of the pressure, density, and temperature of the air through which the vehicle is traveling. Flight testing, therefore, requires precise measurements of these properties in order to provide accurate predictions of aircraft performance. Aircraft air data systems generally include pitot-static systems and total temperature probes which measure free stream static pressure, total pressure, and total temperature. However, these measurements include errors for which corrections must be applied. The purpose of this text is to provide an understanding of pitot-static theory and the process for determining pitot-static and temperature errors through air data system calibration flight testing.

Before discussing pitot-static theory, we should first have a basic understanding of the atmosphere in which we fly. The properties of the atmosphere have a daily, seasonal, and geographic dependence and, in fact, are constantly changing. Solar radiation and weather patterns can cause rapid and significant local variations in the atmosphere. It is not practical to account for these variations when considering the performance of flight vehicles. Therefore, an atmospheric model was developed to provide a common reference based on mean or "standard day" values of the atmospheric properties at each altitude. This model, called the standard atmosphere, allows us to relate theoretical predictions, wind tunnel data, and flight test data. Through it, test data collected on a "non-standard" or "test" day can be corrected to standard day, and flight manual charts can include corrections to predict performance on any given day.

5.2 THE STANDARD ATMOSPHERE

5.2.1 DIVISIONS AND LIMITS OF THE ATMOSPHERE

The atmosphere is divided into four major divisions which are associated with unique physical characteristics. These divisions are the *troposphere*, the *stratosphere*, the *ionosphere*, and the *exosphere*. The boundary between the troposphere and the stratosphere is called the *tropopause*.

The *troposphere* is the division closest to the earth's surface. Its upper limit varies from approximately 28,000 feet (8.5 km) and -46° C at the poles to 56,000 feet (17 km) and -79° C at the equator. These temperatures and altitudes also vary during the day and with the seasons of the year. Additionally, in this region of the atmosphere, temperature decreases with height. This phenomenon results from the fact that the

lower portion of the atmosphere is almost transparent to most short wave radiation received from the sun. Thus, a large portion of the sun's radiation is transmitted to and absorbed by the earth's surface. The region of the troposphere near the earth is heated from below by the long wave radiation from the earth's surface. This radiation in turn heats the rest of the troposphere mainly by convection, conduction, and re-radiation of the long wave energy. Most turbulence is caused by convection or vertical currents, and practically all weather phenomena are contained in this division.

The second major division of the atmosphere is the *stratosphere*. This layer of air extends from the troposphere outward to a distance of approximately 79 km (49 mi). However, the designation for this division has lost some popularity in recent years since the original definition of the stratosphere included constant temperature with height. Recent data have shown that the temperature is constant at approximately 217 K only between about 11 and 20 km (36,089 ft - 65,617 ft or 7 - 12 mi) in altitude; then increases to approximately 270 K at 47 km (29 mi); and finally decreases to approximately 180 K at 79 km. Since these temperature variations are inconsistent with the basic definitions of the stratosphere, some authors have divided this area into two divisions: Stratosphere - 11 to 20 km, and Mesosphere - 20 to 79 km. In the stratosphere, motion is mainly horizontal, and little turbulence is found.

The third major division is the *ionosphere*. It extends between approximately 79 to 480 km (49 - 300 mi). Large numbers of free ions are present in this layer, and a number of electrical phenomena take place in this portion of the atmosphere. The temperature increases with height to a kinetic temperature of 1500 K at 480 km. Low orbiting spacecraft, such as the Space Shuttle, fly within this region.

The fourth major division is the *exosphere*, the outermost layer of the atmosphere. It starts at 480 km and is also characterized by a large number of free ions. In this layer, the air is extremely rare, with density varying by several orders of magnitude, depending upon the activity of the sun and other celestial factors.

5.2.2 DEVELOPMENT OF THE STANDARD ATMOSPHERE[1]

The standard atmosphere models the variation of ambient pressure, P_a, ambient density, ρ_a, and ambient temperature, T_a, with altitude. At the present time there are

[1]Based on a similar discussion in Chapter 3 of Reference 1.

several atmosphere standards which have been established. The most common one in this country is the U.S. Standard Atmosphere - 1962. However, there has been a much greater inventory of experimental data assembled since 1962, over parts of the solar cycle not available for the 1962 Standard Atmosphere. This led to the revision of this model above 51 km to produce the U.S. Standard Atmosphere - 1976 (see Reference 2). The European nations use the ICAO (International Civil Aviation Organization) standard atmosphere. Both the U.S. and European standard atmospheres were developed to approximate the standard average day conditions at 40° to 45° North latitude and are basically the same up to an altitude of approximately 66,000 feet. While the Flight Test Center has adopted the 1976 U.S. Standard Atmosphere as the atmospheric model for data standardization, either of the U.S. 1962 or 1972 atmospheric models are valid for the altitudes flown (0 - 40,000 ft MSL) at the Test Pilot School.

5.2.2.1 GRAVITATIONAL VARIATION WITH ALTITUDE

In order to build a mathematical model of the atmosphere we must first consider the variation of gravity with altitude. From Newton's law of gravitation, the force of gravity, and thus gravitational acceleration, g, varies inversely as the square of the distance from the center of the earth. If we consider imaginary spherical surfaces of constant gravity enclosing the earth, called *geopotential surfaces*, we can write Newton's law in terms of the gravitational acceleration of a geopotential surface located at sea level, g_{SL}, as

$$g = g_{SL}\left(\frac{R_{E_{SL}}}{R_{E_{SL}} + h}\right)^2 \tag{5.1}$$

where $R_{E_{SL}}$ is the radius of the sea level geopotential surface, *h is the geometric distance above this surface*, and, by definition,

$$g_{SL} = 32.174 \text{ ft/sec}^2 = 9.807 \text{ m/s}^2$$

Since the earth is neither a perfect spheroid nor homogeneous, the radius of the geopotential surfaces vary with the Earth's latitude and longitude. Thus, the value of the radius of the sea level geopotential surface will depend on the specific location on the Earth's surface. At the end of Edwards runway 22, the value is:

$$R_{E_{SL}} = 20{,}902{,}690.55 \text{ ft}$$

5.2.2.2 THE HYDROSTATIC EQUATION

The foundation of the standard atmosphere is the hydrostatic equation, which results from a simple force balance on an element of fluid at rest. Consider the small fluid element of height *dh* shown in Figure 5.1. The element upper and lower surfaces are each of surface area *dA*. On the bottom face, a pressure *P* is felt, which gives rise to an upward force of *P dA*. The top face is slightly higher in altitude, therefore since pressure varies with altitude it will differ slightly from the bottom face by a differential amount *dP*. Thus, a downward force of *(P+dp) dA* is felt by the upper face. The only remaining force is the weight of the fluid element, *dm g*, where *dm* is the mass of the fluid element and *g* is the local gravitational acceleration. But the fluid element mass is simply the fluid density ρ (assumed constant for the element) times the volume of the element *dV* (= *dh dA*). Thus, summing the forces, we find:

$$P dA - (P+dP)\, dA - \rho\, dV\, g = 0$$

Solving for the change in pressure with altitude results in the general form of the hydrostatic equation

$$dP = -\rho\, g\, dh \qquad (5.2)$$

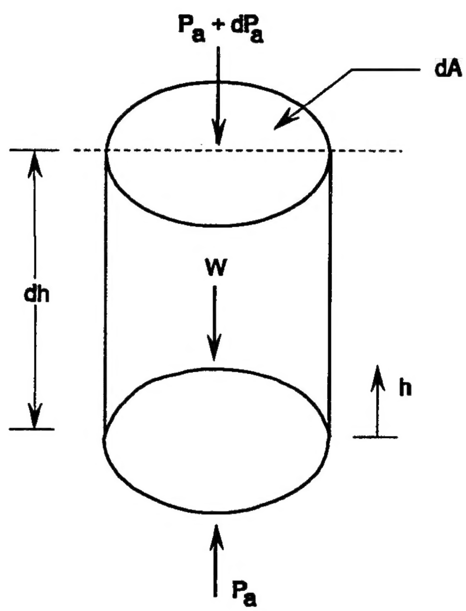


Figure 5.1 Forces Acting on a Vertical Column of Air

5.2.2.3 GEOPOTENTIAL ALTITUDE

We can define a new energy term, called geopotential, to remove the dependence of the hydrostatic equation on gravity. Geopotential is a measure of the gravitational potential energy of a unit mass at a point relative to mean sea level. It is defined in differential form by the equation

$$g\,dh = g_{SL}\,dH \tag{5.3}$$

where *H is the geopotential altitude* and has units of geopotential feet. In a physical sense, geopotential is equivalent to the work done in elevating a unit mass from sea level to a tapeline altitude expressed in feet. Geopotential altitude can be thought of as a fictitious altitude which is physically compatible with the assumption:

$$g = const = g_{SL}$$

Substituting equation 5.3 into equation 5.2 yields a new final hydrostatic equation in terms of geopotential altitude and the sea level gravitational constant:

$$dP = -\,\rho\,g_{SL}\,dH \tag{5.4}$$

We can also substitute equation 5.1, gravity as a function of geometric altitude, into equation 5.3 and easily integrate to produce the following relationship between geometric and geopotential altitude:

$$H = \left(\frac{R_{E_{SL}}}{R_{E_{SL}} + h}\right) h \tag{5.5}$$

From this equation, it can be shown that for most purposes, errors introduced by assuming geometric and geopotential altitudes are equal in the troposphere are insignificant. At about 20,000 ft (6.1 km), the error is less than 0.1 %. Even at the upper limit of the stratosphere (82,021 ft or 25 km), the error is less than 0.4 %. Only at altitudes above 211,138 ft (64.4 km) does the difference exceed 1.0 %. *Therefore, we can substitute dh for dH in equation 5.4 above with minimal error when analyzing flight within the lower atmosphere.*

5.2.2.4 DEFINITION OF THE STANDARD ATMOSPHERE

The definition of the standard atmosphere is based on the following assumptions:

1. The air is dry.
2. The atmosphere is a perfect gas which obeys the equation of state:

$$P = \rho R T \tag{5.6}$$

where[2]

R ≡ dry air gas constant (= 3089.8 $ft^2/K\ sec^2$ or 1716.5 $ft^2/R\ sec^2$)

T ≡ absolute temperature in R or K

The equation of state closely describes the behavior of the atmosphere in the lower layers and is adequate for the portion of the atmosphere where aircraft performance data are of interest. However, when the number of molecules in a given area are reduced to a point where uniform pressure no longer exists (therefore, intermolecular forces cannot be considered negligible), the equation of state given above is no longer valid. Because of this rarity of the atmosphere and the change in the mean molecular weight of the air due to disassociation, we will consider the equation of state only valid up to an altitude of about 290,000 ft (88 km).

3. Temperature variation is a function of geopotential altitude and consists of a series of straight line segments, as illustrated in Figure 5-2. This variation is based on experimental evidence and includes constant temperature (isothermal) regions and constant lapse rate (gradient) regions.

From the basic assumptions for the standard atmosphere listed above, it is possible to derive the relationships for standard day (SD) ambient temperature, $T_{a\,SD}$, ambient pressure, $P_{a\,SD}$, and ambient density, $\rho_{a\,SD}$, as functions of geopotential altitude. Dividing the hydrostatic equation (equation 5.4) by the equation of state (equation 5.6) produces the general differential equation of the standard atmosphere:

$$\frac{dP}{P} = -\frac{\rho\, g_{SL}\, dH}{\rho R T} = -\frac{g_{SL}}{RT}\, dH \qquad (5.7)$$

First consider the *isothermal regions*. The pressure, density, and temperature at the base of this region are given by P_B, ρ_B, and T_B, respectively. Thus, since the temperature is constant,

$$T_{a_{SD}} = T_B \qquad (5.8)$$

The pressure as a function of H can be found from integration from the base of the particular region to some point within that region. Thus, in the isothermal regions,

[2] Recall the conversion relations to transform temperature expressed in degrees Fahrenheit (°F) or degrees Celsius (°C) to absolute temperature:

$$T(K) = T(°C) + 273.15$$
$$T(R) = T(°F) + 459.67$$

Additionally, the conversion between absolute temperature scales is:

$$T(R) = 1.8\ T(K)$$

$$\int_{P_B}^{P_{a_{SD}}} \frac{dP}{P} = -\frac{g_{SL}}{RT_B}\int_{H_B}^{H} dH$$

Solving results in the following equation for the standard day ambient pressure:

$$\frac{P_{a_{SD}}}{P_B} = e^{-(g_{SL}/RT_B)\,(H-H_B)} \tag{5.9}$$

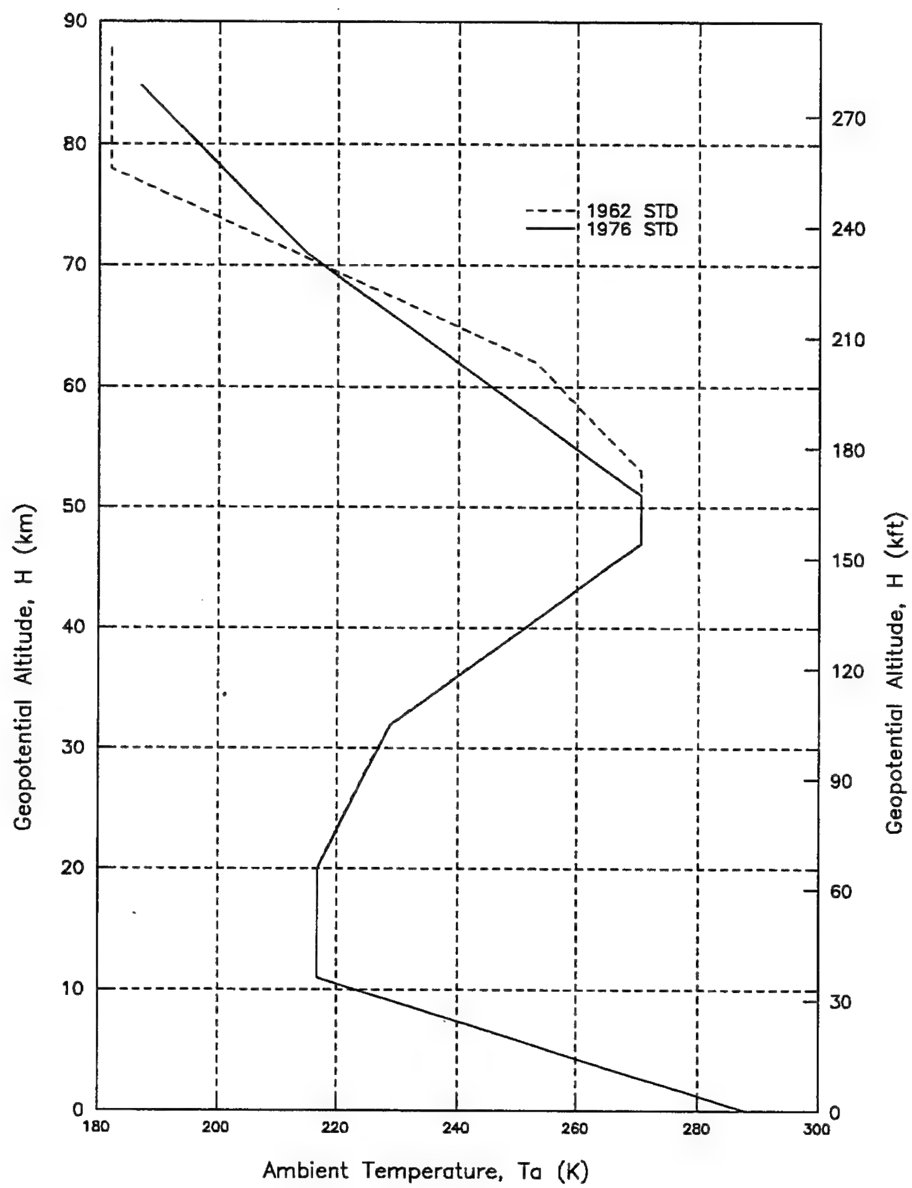


Figure 5.2 1962 and 1976 U.S. Standard Atmosphere Lapse Rates

From the equation of state, again noting that temperature is constant in the region ($T_{a_{SD}} = T_B$):

$$\frac{P_{a_{SD}}}{P_B} = \frac{\rho_{a_{SD}} T_B}{\rho_B T_B} = \frac{\rho_{a_{SD}}}{\rho_B}$$

Substitution of this equation into equation 5.9 produces the final required relation within the isothermal regions for the ambient density

$$\frac{\rho_{a_{SD}}}{\rho_B} = e^{-(g_{SL}/RT_B)(H-H_B)} \tag{5.10}$$

Now consider the *gradient regions*. For these regions, the temperature variation with geopotential altitude, defined as the *temperature lapse rate*, *L*, is assumed to be constant. Thus,

$$L = \frac{dT}{dH} \equiv \text{constant} \tag{5.11}$$

Integrating this equation,

$$L \int_{H_B}^{H} dH = \int_{T_B}^{T_{a_{SD}}} dT$$

results in an expression for ambient temperature as a function of geopotential altitude within this region is

$$\frac{T_{a_{SD}}}{T_B} = 1 + \frac{L}{T_B}(H - H_B) \tag{5.12}$$

Substituting equation 5.11 into the equation 5.7 provides a differential equation for the pressure variation with geopotential altitude:

$$\int_{P_B}^{P_{a_{SD}}} \frac{dP}{P} = -\frac{g_{SL}}{LR} \int_{T_B}^{T_{a_{SD}}} \frac{dT}{T}$$

Integrating this equation and then substituting equation 5.12 results in the following relation for ambient pressure as a function of geopotential altitude for the gradient region:

$$\frac{P_{a_{SD}}}{P_B} = \left(1 + \frac{L}{T_B}(H - H_B)\right)^{-g_{SL}/LR} \tag{5.13}$$

Again, from the equation of state

$$\frac{P_{a_{SD}}}{P_B} = \frac{\rho_{a_{SD}} T_{a_{SD}}}{\rho_B T_B}$$

Substitution into equation 5.13 yields the equation for ambient density as a function of geopotential altitude within the gradient region:

$$\frac{\rho_{a_{SD}}}{\rho_B} = \left(1 + \frac{L}{T_B}\,(H - H_B)\right)^{-\left(\frac{g_{SL}}{LR} + 1\right)} \tag{5.14}$$

5.2.3 THE 1962 AND 1976 U.S. STANDARD ATMOSPHERE

Each standard atmosphere model defines the geopotential altitude bands for each atmospheric region and the associated atmospheric properties of each of these regions. The atmospheric properties are generally provided in terms of the ratio of the particular parameter with respect to its sea level value, as follows:

1. Temperature Ratio: $\theta = \dfrac{T_a}{T_{a_{SL}}}$ (5.15)

2. Pressure Ratio: $\delta = \dfrac{P_a}{P_{a_{SL}}}$ (5.16)

3. Density Ratio: $\sigma = \dfrac{\rho_a}{\rho_{a_{SL}}} = \dfrac{\delta}{\theta}$ (From the Equation of State) (5.17)

For either of the 1962 or 1976 U.S. Standard Atmospheres, the geopotential altitude bands and properties of the regions of interest are as follows:

1. Troposphere

    Altitude band: $-16{,}404.2 \le H \le 36{,}089.24$ geopotential ft

    Base Altitude: 0 geopotential ft

    Base Properties:

    $T_B = T_{a_{SL}} = 15°C$ or 288.15 K

    $P_B = P_{a_{SL}} = 29.9213$ in Hg or 2116.22 lb/ft^2

    $\rho_B = \rho_{a_{SL}} = 0.0023769$ sl/ft^3

    $L = -\,0.0019812$ K/ft

From equations 5.12 - 5.14 derived for the gradient temperature profile, the standard day properties are:

$$\theta_{SD} = \frac{T_{a_{SD}}}{T_{a_{SL}}} = 1 - K_1 H \tag{5.18}$$

$$\delta_{SD} = \frac{P_{a_{SD}}}{P_{a_{SL}}} = (1 - K_1 H)^{K_2} \tag{5.19}$$

$$\sigma_{SD} = \frac{\rho_{a_{SD}}}{\rho_{a_{SL}}} = (1 - K_1 H)^{K_2 - 1} \tag{5.20}$$

where

$$K_1 = -\frac{L}{T_{a_{SL}}} = 6.87558\text{x}10^{-6}/\text{geo ft}$$

$$K_2 = -\frac{g_{SL}}{RL} = 5.25590$$

2. Stratosphere

Altitude band: 36,089.24 < H ≤ 65,616.80 geopotential ft

Base Altitude: 36,089.24 geopotential ft

Base Properties:

T_B = 216.65 K

P_B = 472.679 lb/ft^2

ρ_B = 0.0007061 sl/ft^3

L = 0 K/ft

From equations 5.8 - 5.10 for the constant temperature regions, the standard day properties are:

$$\theta_{SD} = \frac{T_{a_{SD}}}{T_{a_{SL}}} = 0.751865 \tag{5.21}$$

$$\delta_{SD} = \frac{P_{a_{SD}}}{P_{a_{SL}}} = 0.223360\, e^{-K_3 (H - 36089.24 \text{ geo ft})} \tag{5.22}$$

$$\sigma_{SD} = \frac{\rho_{a_{SD}}}{\rho_{a_{SL}}} = 0.297075\, e^{-K_3 (H - 36089.24 \text{ geo ft})} \tag{5.23}$$

where

$$K_3 = \frac{g_{SL}}{R T_{a_{SL}}} = 4.80637\text{x}10^{-5}/\text{geo ft}$$

5.3 ALTITUDE MEASUREMENT

There are several different means by which to determine altitude. Radar or triangulation can be used to determine either geometric altitude, h, or the distance above the local ground level, called tapeline altitude, h_{tape}. If a highly sensitive accelerometer could be developed to measure the force of gravity, it could provide geometric altitude in feet based on equation 5.1. But this device would give the correct reading only when in level unaccelerated flight. In actual practice, however, the geometric or tapeline altitudes are only important for takeoff and obstacle clearance tests, terrain following, or trajectory determination. Other types of altitude could be determined based on measuring the atmospheric properties and using the standard day relationships. These altitudes would only correspond to geometric altitudes on a standard day. A temperature altitude can be obtained by taking a temperature gauge and modifying it to read in feet for a corresponding temperature as determined from equations 5.18 or 5.21. Since temperature inversions and nonstandard lapse rate are the rule rather than the exception, and temperature changes greatly between day and night, with the seasons of the year, and with latitude, such a technique would yield highly inaccurate results. If some instrument were available to measure density, the same type of technique could be employed, and density altitude could be determined from equations 5.20 or 5.23. However, density is also affected by the temperature inversions and would again provide poor results. The remaining atmospheric parameter, pressure, is not greatly affected by temperature inversions making it the most practical of the three for in-flight altitude measurement. An instrument, called the altimeter, has been developed which senses the ambient pressure and indicates the corresponding standard altitude based on equations 5.19 or 5.22. This altitude is called *pressure altitude* and is the parameter on which most performance and flying qualities flight testing is based.

5.3.1 THE ALTIMETER

Most altitude measurements are made with a sensitive absolute pressure gauge, or an altimeter, scaled so that a pressure decrease indicates an altitude increase in accordance with the standard atmosphere, as illustrated in Figure 5.3 (Reference 3). Figure 5.4 shows that the heart of the altimeter is an evacuated metal bellows which expands or contracts with changes in outside pressure. The bellows is connected to a series of gears and levers which cause a pointer to move in response to the changes in the bellows' volume. It is through this network that equations 5.19 and 5.22 are mechanized. The entire device is placed in an airtight case which is vented to a static

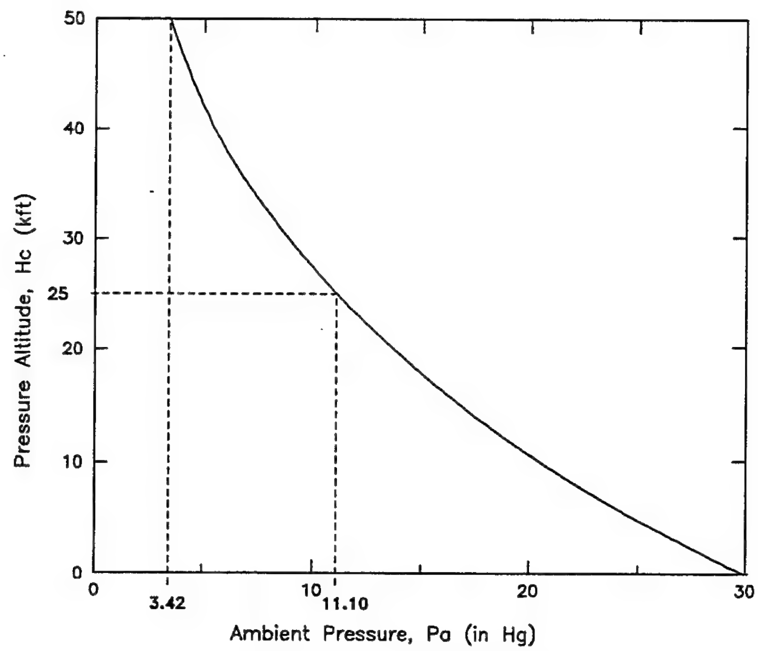


Figure 5.3 Standard Day Pressure Variation with Altitude

source; therefore the indicator reads the pressure supplied to the case. A provision has been made in the construction of the altimeter in the form of an altimeter setting, called the Kollsman window, whereby the scale reading can be adjusted up or down (the same as moving the curve of Figure 5.3 vertically by changing the sea level

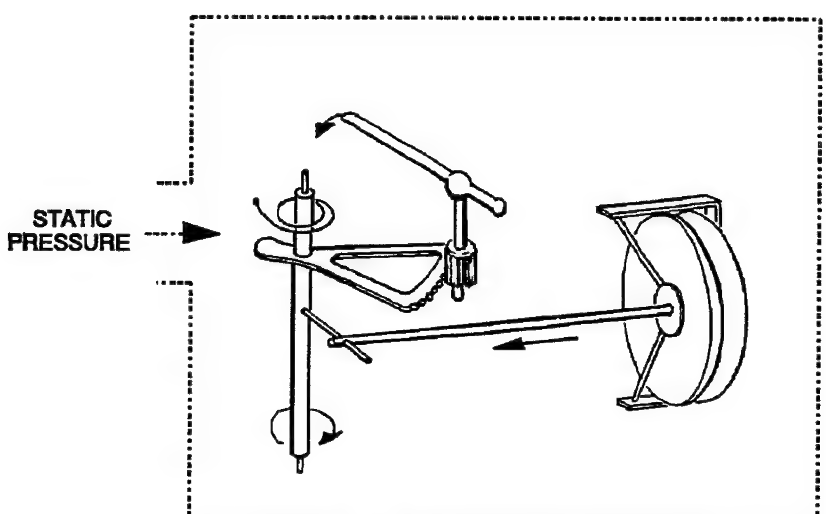


Figure 5.4 Altimeter Schematic

pressure constant, $P_{a_{SL}}$). This will allow a pilot to reset the altimeter so that it will read the true local runway elevation upon landing. *Only when the altimeter setting is 29.92 in Hg does it provide altitude readings in accordance with the standard atmosphere equations.*

5.3.2 NON-STANDARD DAY PRESSURE VARIATION WITH ALTITUDE

It should be obvious that altimeters will not provide correct altitude readings unless the pressure at altitude is the same as that defined for a standard day. On non-standard or "test" days, the actual pressure profile will shift due to weather patterns or temperature profile deviation from standard day, as illustrated in Figure 5.5. If an aircraft is flying at a particular geopotential altitude on a test day with a warmer than standard day temperature, the altimeter will sense the corresponding pressure as shown by the vertical dashed line. However, since the altimeter is built around the standard curve (shown by the solid curve), the instrument will indicate a different altitude, defined as *pressure altitude* (or corrected altitude), H_c, assuming an altimeter setting of 29.92 in Hg. *Only on a theoretical standard day will pressure altitude correspond to geopotential altitude.* Therefore we must rewrite equations 5.19 and

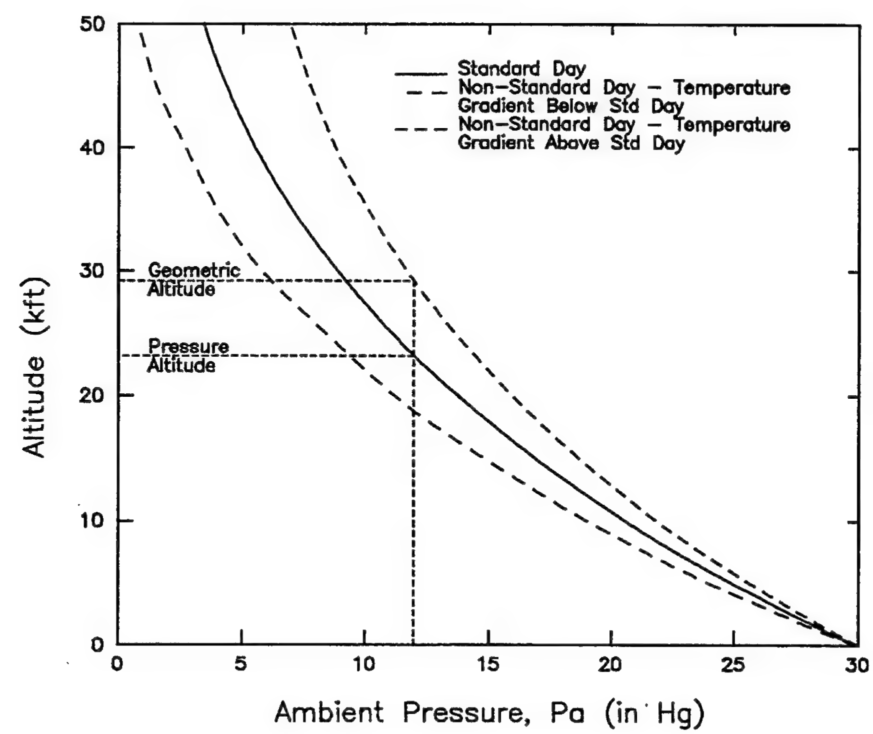


Figure 5.5 Non-Standard Day Pressure Profile Comparison

5.22 in terms of pressure altitude instead of geopotential altitude, forming the following two altimeter equations:

a. Troposphere ($-16{,}404.2 \le H_c \le 36{,}089.24$ ft)

$$\delta \equiv \frac{P_a}{P_{a_{SL}}} = (1 - 6.87558\text{x}10^{-6}\, H_c)^{5.2559} \tag{5.24}$$

b. Stratosphere ($36{,}089.24 < H_c \le 65{,}616.80$ ft)

$$\delta \equiv \frac{P_a}{P_{a_{SL}}} = 0.223360\ e^{-4.80637\text{x}10^{-5}\,(H_c - 36089.24)} \tag{5.25}$$

In order to relate pressure altitude to geopotential altitude on a test day, the test day temperature profile must be known for use in integrating equation 5.7. Since measuring this profile is impractical, either radar or triangulation must be used to find the actual geometric altitude. However, a relationship between small pressure altitude changes and geopotential altitude changes can be found. Replacing the differentials in equation 5.7 with small perturbations, we find on a *standard day* that

$$\Delta P \cong -\frac{P_a g_{SL}}{R T_{a_{SD}}} \Delta H = -\frac{P_a g_{SL}}{R T_{a_{SD}}} \Delta H_c \tag{5.26}$$

noting that pressure altitude and geometric altitude are exactly the same on a standard day. For the same measured ambient pressure on a *test day*, the equation for the small change in pressure would be

$$\Delta P \cong -\frac{P_a g_{SL}}{R T_a} \Delta H \tag{5.27}$$

where T_a is the test day ambient temperature. Thus, for a given change in pressure, we can equate the standard day and test day expressions

$$-\frac{P_a g_{SL}}{R T_{a_{SD}}} \Delta H_c = -\frac{P_a g_{SL}}{R T_a} \Delta H$$

resulting in a final relationship between geopotential and pressure altitude:

$$\Delta H = \frac{T_a}{T_{a_{SD}}} \Delta H_c \tag{5.28}$$

The forces acting on an aircraft in flight are directly dependent upon air density. Thus, in theory, density altitude is the independent variable which should be used for aircraft performance comparisons. However, from a practical standpoint, since density is determined by pressure and temperature through the equation of state, *pressure altitude is used as the independent variable with test day data corrected for non-standard temperature*. This greatly facilitates flight testing since the test pilot

can maintain a given pressure altitude regardless of what the test day conditions are. Twenty-thousand feet pressure altitude is the same from one day to the next and from one aircraft to another. By applying a correction for non-standard temperature, performance data can be presented for any day conditions.

5.4 AIRSPEED MEASUREMENT

Airspeed system theory was first developed with the assumption of incompressible flow. This assumption is only useful for airspeeds of 250 knots or less ($M < 0.3$) at relatively low altitudes. However, since various concepts and nomenclature of incompressible flow are still in use, it is examined here as a step toward the understanding of compressible flow relations.

5.4.1 INCOMPRESSIBLE FLOW

Airspeed theory for all types of flow stems from Euler's equation:

$$dP + \rho V\, dV = 0 \tag{5.29}$$

Here we have neglected gravity and assumed that no energy is being added to or subtracted from the flow. For the incompressible case, density is assumed constant. Integrating this equation yields Bernoulli's equation for incompressible flow:

$$P + \frac{1}{2}\rho V^2 = C \tag{5.30}$$

where C is a constant. Thus, if we consider an aircraft pitot-static system as shown in Figure 5.6, application of equation 5.30 results in:

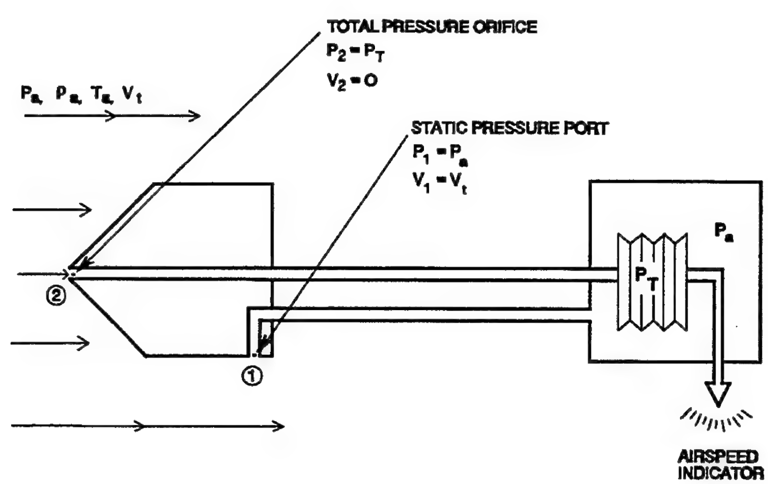


Figure 5.6 Pitot Static Schematic

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

where P_1 and V_1 represent the pressure and speed measured at the static port and P_2 and V_2 represent the pressure and speed in the total pressure probe. Since the air within the total pressure probe has been brought to rest, $V_2 = 0$, resulting in

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2$$

Changing subscripts to reflect ambient and total pressures and noting that V_1 is the true airspeed relative to the surrounding air mass, V_t, yields

$$P_a + \frac{1}{2}\rho_a V_t^2 = P_T$$

Solving for the true airspeed for *incompressible flow*,

$$V_t = \sqrt{\frac{2\,(P_T - P_a)}{\rho_a}} \tag{5.31}$$

5.4.2 COMPRESSIBLE SUBSONIC FLOW

Euler's equation is also used to develop the airspeed equations for compressible flow. However, in this case, density can no longer be assumed constant. In the *subsonic* case, we can assume the flow is *isentropic*. Recall that an isentropic process is one in which there is no heat interaction (adiabatic) and the effects of friction can be ignored (reversible). The derivation of the basic subsonic isentropic flow relations was presented in previous subsonic and supersonic aerodynamics courses and can be reviewed in Chapter 4 of Reference 1. Starting with the total pressure relation

$$\frac{P_T}{P_a} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{(\gamma-1)}} \tag{5.32}$$

where

P_T ≡ freestream total pressure
P_a ≡ freestream ambient pressure
M ≡ freestream Mach number
γ ≡ ratio of specific heats (c_p/c_v)
c_p ≡ specific heat at constant pressure
c_v ≡ specific heat at constant volume

we can derive the subsonic Mach number relation by simply solving for Mach number and rearranging:

$$M^2 = \frac{2}{\gamma - 1}\left[\left(\frac{P_T - P_a}{P_a} + 1\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right] \tag{5.33}$$

For air at normal conditions (0-60°C), $\gamma = 1.40$. Substituting:

$$M^2 = 5\left[\left(\frac{P_T - P_a}{P_a} + 1\right)^{\frac{2}{7}} - 1\right] \tag{5.34}$$

Substituting the equation of state (5.6) into the equation for the speed of sound, a, we find:

$$a = \sqrt{\gamma R T_a} = \sqrt{\gamma \frac{P_a}{\rho_a}} \quad \left(= \sqrt{\gamma R T_{a_{SL}}}\,\sqrt{\theta}\right) \tag{5.35}$$

The Mach number is defined as the ratio of true airspeed to the speed of sound. Therefore, substituting equation 5.35, we can write

$$M^2 = \left(\frac{V_t}{a}\right)^2 = \frac{V_t^2}{\gamma \dfrac{P_a}{\rho_a}} \tag{5.36}$$

Substitution into equation 5.33 yields the compressible, subsonic relation for true airspeed

$$V_t^2 = \frac{2\gamma P_a}{\rho_a(\gamma - 1)}\left[\left(\frac{P_T - P_a}{P_a} + 1\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right] \tag{5.37}$$

Again, substituting for γ, this equation reduces to

$$V_t^2 = \frac{7 P_a}{\rho_a}\left[\left(\frac{P_T - P_a}{P_a} + 1\right)^{\frac{2}{7}} - 1\right] \tag{5.38}$$

5.4.3 COMPRESSIBLE SUPERSONIC FLOW

If the flow is supersonic, it must pass through a shock wave in order to slow to stagnation conditions. There is a loss of total pressure when the flow passes through the shock wave. Thus, the pitot probe does not measure the total pressure of the supersonic freestream. The solution for supersonic flight is derived by considering a normal shock compression in front of the total pressure tube and an isentropic compression in the subsonic region aft of the shock as shown in Figure 5.7. The normal shock assumption is good, since the pitot tube has a small frontal area. Consequently, the radius of the shock in front of the hole may be considered infinite. From Figure 5.7, behind the shock, equation 5.32 can be rewritten as

$$\frac{P_T'}{P_a'} = \left(1 + \frac{\gamma - 1}{2} M'^2\right)^{\frac{\gamma}{(\gamma-1)}} \tag{5.39}$$

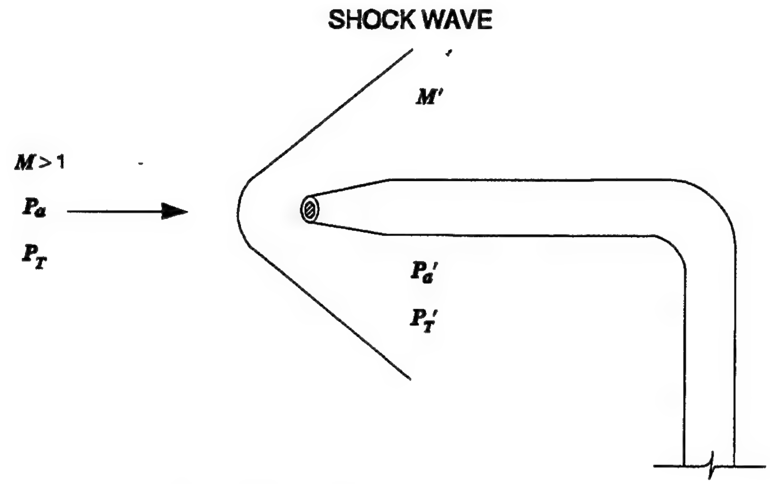


Figure 5.7 Pitot Tube in Supersonic Flow

The Mach number drop across the shock is given by

$$M'^2 = \frac{(\gamma-1)M^2 + 2}{2\gamma M^2 - (\gamma-1)} \tag{5.40}$$

and the ratio of static pressures across the shock is given by

$$\frac{P_a'}{P_a} = \frac{1-\gamma+2\gamma M^2}{1+\gamma} \tag{5.41}$$

Multiplying equation 5.39 by equation 5.41 and substituting equation 5.40 produces:

$$\frac{P_T' - P_a}{P_a} = \left(\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2(\gamma-1)}\right)^{\frac{\gamma}{(\gamma-1)}} \frac{1-\gamma+2\gamma M^2}{\gamma+1} - 1 \tag{5.42}$$

Substituting for γ, this equation reduces to:

$$\frac{P_T' - P_a}{P_a} = \left(\frac{36\ M^2}{5[7M^2-1]}\right)^{3.5}\left(\frac{7M^2-1}{6}\right) - 1$$

$$= \frac{166.921\,M^7}{[7M^2-1]^{2.5}} - 1 \tag{5.43}$$

The resulting equation 5.42 is known as the Rayleigh Supersonic Pitot tube formula, which relates the total pressure behind the shock, P_T', to the free stream ambient pressure, P_a, and the free stream Mach number. Equations 5.42 and 5.43 can be solved iteratively for Mach number, therefore they represent the desired compressible, supersonic Mach number relations. By substituting the definition of Mach number into these equations, we find the supersonic true airspeed relations:

$$\frac{P_T' - P_a}{P_a} = \left(\frac{(\gamma+1)^2\left(\dfrac{V_t}{a}\right)^2}{4\gamma\left(\dfrac{V_t}{a}\right)^2 - 2(\gamma-1)}\right)^{\frac{\gamma}{(\gamma-1)}} \frac{1-\gamma+2\gamma\left(\dfrac{V_t}{a}\right)^2}{\gamma+1} - 1 \tag{5.44}$$

Substituting for γ produces

$$\frac{P_T' - P_a}{P_a} = \left(\frac{36\left(\dfrac{V_t}{a}\right)^2}{5\left[7\left(\dfrac{V_t}{a}\right)^2 - 1\right]}\right)^{3.5} \left(\frac{7\left(\dfrac{V_t}{a}\right)^2 - 1}{6}\right) - 1$$

$$= \frac{166.921\left(\dfrac{V_t}{a}\right)^7}{\left[7\left(\dfrac{V_t}{a}\right)^2 - 1\right]^{2.5}} - 1 \tag{5.45}$$

Notice that all of the above equations specifically included the term $P_T' - P_a$. This is due to the fact that it represents the differential pressure felt by the airspeed indicator diaphragm and, therefore, has been given the name differential pressure or q-compressible, q_c. In the subsonic case, since there is no shock present;

$$P_T = P_T'$$

Thus, the supersonic and subsonic definitions of differential pressure are:

Supersonic Flow: $$q_c = P_T' - P_a \tag{5.46}$$

Subsonic Flow: $$q_c = P_T - P_a \tag{5.47}$$

It is important to understand the difference between the differential pressure, q_c, and the dynamic pressure, q. Dynamic pressure is defined as

$$q = \frac{1}{2}\rho V_t^2 \tag{5.48}$$

Only under conditions of low altitude and low speed do the two terms converge. This can be seen in the series expansion of the relationship between them

$$q_c = q\left(1 + \frac{M^2}{4} + \frac{M^4}{40} + \frac{M^6}{16000} + \ldots\right) \tag{5.49}$$

As Mach number becomes small, this series converges rapidly. Obviously, by inspection of equation 5.31 for incompressible flow, the differential and dynamic pressures are equal.

5.4.4 EQUIVALENT AND CALIBRATED AIRSPEEDS

It is possible to use a pitot-static system and build an airspeed indicator to conform to the incompressible and compressible true airspeed relations presented thus far. However, there are disadvantages in doing so.

1. The subsonic relations include differential pressure, ambient pressure, and density. This adds to the complexity of the instrument, which would require two bellows, one for measuring differential pressure, and the other for ambient pressure, as well as a temperature probe (since ambient density can be calculated from the equation of state using ambient pressure and temperature).

2. Except for navigation, an instrument providing true airspeed would not give the pilot information he requires. For example, when landing, the aircraft should be flown near the maximum lift coefficient, $C_{L_{max}}$, a fixed amount before stall. Thus,

$$L = C_{L_{max}} S \frac{1}{2} \rho_a V_{t_{Land}}^2 \approx W$$

where

L ≡ aircraft lift force
W ≡ aircraft weight
S ≡ reference wing area

resulting in

$$V_{t_{Land}} = \sqrt{\frac{2W}{C_{L_{max}} S \rho_a}}$$

This means that a pilot would be required to compute a different landing speed for each combination of weight and ambient density.

5.4.4.1 EQUIVALENT AIRSPEED

Density is really the variable which causes the deficiencies in a true airspeed indicator. A solution to the problem is to somehow remove the dependence of this equation on density. By replacing ρ with $\rho_{a_{SL}}$ in Equation 5.31, the resulting combination velocity-density term is defined as equivalent airspeed, V_e. Thus for *incompressible subsonic flow:*

$$V_e = \sqrt{\frac{2(P_T - P_a)}{\rho_{a_{SL}}}} = \sqrt{\frac{2q_c}{\rho_{a_{SL}}}} \tag{5.50}$$

where

$$V_e \equiv V_t \sqrt{\frac{\rho_a}{\rho_{a_{SL}}}} = V_t \sqrt{\sigma} \tag{5.51}$$

Equation 5.50 for the incompressible subsonic equivalent airspeed was used in airspeed indicators primarily before World War II. However, as aircraft speed and altitude capabilities increased the error resulting from the assumption that density remains constant became significant. Airspeed indicators for today's aircraft must be built to consider compressibility, as provided in the following compressible relations.

For *subsonic compressible flow*, substituting the definition of equivalent airspeed presented in equation 5.51 and the definition of differential pressure into equations 5.37 and 5.38 yields:

$$V_e = \left[\frac{2\gamma P_a}{\rho_{a_{SL}}(\gamma-1)} \left[\left(\frac{q_c}{P_a} + 1 \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right] \right]^{\frac{1}{2}} \tag{5.52}$$

and (for $\gamma = 1.4$)

$$V_e = \left[\frac{7 P_a}{\rho_{a_{SL}}} \left[\left(\frac{q_c}{P_a} + 1 \right)^{\frac{2}{7}} - 1 \right] \right]^{\frac{1}{2}} \tag{5.53}$$

We can rewrite equation 5.51 in terms of Mach number and the pressure ratio:

$$V_e = V_t \sqrt{\sigma} = M a \sqrt{\sigma} = M a_{SL} \sqrt{\sigma \theta} = M a_{SL} \sqrt{\delta}$$

where

$$a_{SL} \equiv 661.48 \text{ kt}$$

Substituting into equations 5.44 and 5.45 yields the relation for equivalent airspeed in the *supersonic case*

$$\frac{q_c}{P_a} = \left(\frac{(\gamma+1)^2 \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2}{4\gamma \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2 - 2(\gamma-1)} \right)^{\frac{\gamma}{(\gamma-1)}} \left(\frac{1 - \gamma + 2\gamma \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2}{\gamma+1} \right) - 1 \tag{5.54}$$

and (for $\gamma = 1.4$)

$$\frac{q_c}{P_a} = \left(\frac{36 \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2}{5 \left[7 \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2 - 1 \right]} \right)^{3.5} \left(\frac{7 \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2 - 1}{6} \right) - 1$$

$$= \frac{166.921 \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^7}{\left[7 \left(\frac{V_e}{a_{SL}\sqrt{\delta}} \right)^2 - 1 \right]^{2.5}} - 1 \tag{5.55}$$

Considering only the incompressible flow equation for equivalent airspeed (equation 5.50), it is now possible to build a simple airspeed indicator which must measure only differential pressure, q_c. Such a device requires only one bellows and has the following advantages:

1. Because of its simplicity, it has a high degree of accuracy.

2. The indicator is easy to calibrate.

3. V_e is a measured quantity of use to the pilot. Using the landing case

$$W = C_{L_{max}} S \frac{1}{2} \rho_a V_{t_{Land}}^2 = C_{L_{max}} S \frac{1}{2} \rho_{a_{SL}} V_{e_{Land}}^2$$

yielding

$$V_{e_{Land}} = \sqrt{\frac{2W}{C_{L_{max}} S \rho_{a_{SL}}}}$$

Thus, in computing either landing or stall speed, the pilot need only consider one factor - weight.

4. Equivalent airspeed does not vary with temperature or density. Thus, for a given value of q_c, the equivalent airspeed is the same on either a test or standard day.

5.4.4.2 CALIBRATED AIRSPEED

For compressible flow, the equivalent airspeed is a function of both q_c and P_a. If we wish to remove the dependence of this equation on P_a (thereby reducing the number of bellows required for an airspeed indicator), we can define a new term, called calibrated airspeed, V_c, by simply replacing P_a with $P_{a_{SL}}$. Thus, from equations 52 and 53 for *subsonic compressible flow*:

$$V_c = \left[\frac{2\gamma P_{a_{SL}}}{\rho_{a_{SL}} (\gamma - 1)} \left[\left(\frac{q_c}{P_{a_{SL}}} + 1 \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right] \right]^{\frac{1}{2}} \tag{5.56}$$

and (for $\gamma = 1.4$)

$$V_c = \left[\frac{7 P_{a_{SL}}}{\rho_{a_{SL}}} \left[\left(\frac{q_c}{P_{a_{SL}}} + 1 \right)^{\frac{2}{7}} - 1 \right] \right]^{\frac{1}{2}} \tag{5.57}$$

Substituting into equations 5.54 and 5.55 provides the relation for calibrated airspeed in the *supersonic case*

$$\frac{q_c}{P_{a_{SL}}} = \left(\frac{(\gamma+1)^2 \left(\frac{V_c}{a_{SL}}\right)^2}{4\gamma\left(\frac{V_c}{a_{SL}}\right)^2 - 2(\gamma-1)} \right)^{\frac{\gamma}{(\gamma-1)}} \frac{1-\gamma+2\gamma\left(\frac{V_c}{a_{SL}}\right)^2}{\gamma+1} - 1 \tag{5.58}$$

and (for $\gamma = 1.4$)

$$\frac{q_c}{P_{a_{SL}}} = \left(\frac{36\left(\frac{V_c}{a_{SL}}\right)^2}{5\left[7\left(\frac{V_c}{a_{SL}}\right)^2 - 1\right]} \right)^{3.5} \left(\frac{7\left(\frac{V_c}{a_{SL}}\right)^2 - 1}{6} \right) - 1$$

$$= \frac{166.921\left(\frac{V_c}{a_{SL}}\right)^7}{\left[7\left(\frac{V_c}{a_{SL}}\right)^2 - 1\right]^{2.5}} - 1 \tag{5.59}$$

Instruments designed to follow these equations have the following advantages:

1. The indicator is simple in design and construction, accurate, and easy to calibrate.

2. V_c is of direct use to the pilot. It is analogous to V_e in the incompressible case, since at low airspeeds and moderate altitudes $V_e = V_c$. The aircraft stall speed, landing speed, and handling characteristics are proportional to calibrated airspeed for a given gross weight.

3. Since a temperature or density term is not present in the equation for calibrated airspeed, a given value of q_c has the same significance on any day.

5.4.5 THE AIRSPEED INDICATOR

Airspeed indicators are constructed and calibrated according to the calibrated airspeed equations (equations 5.57 and 5.59). In operation, the airspeed indicator is similar to the altimeter, but instead of being evacuated, the inside of the bellows or capsule is connected to the total pressure source, and the case to the static pressure source as shown in Figure 5.8. The instrument then senses total pressure (P_T or P_T') within the capsule and static pressure outside it.

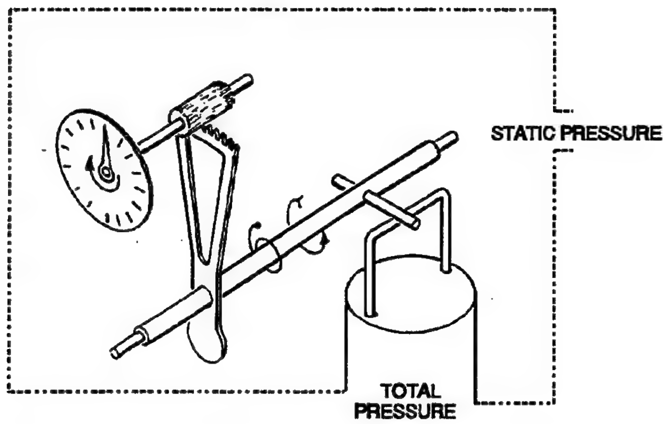


Figure 5.8 Airspeed Indicator Schematic

5.4.6 THE MACHMETER

The Machmeter is essentially a combination altimeter and airspeed indicator designed to solve equations 5.34 and 5.43. An altimeter capsule and an airspeed capsule simultaneously supply signals to a series of gears and levers to produce the Mach indication, as shown in Figure 5.9. Since the construction of the Machmeter requires two bellows, one for q_c and another for P_a, it is complex, difficult to calibrate, and somewhat inaccurate. As a result, Machmeters of this design are used only as a reference in flight test work.

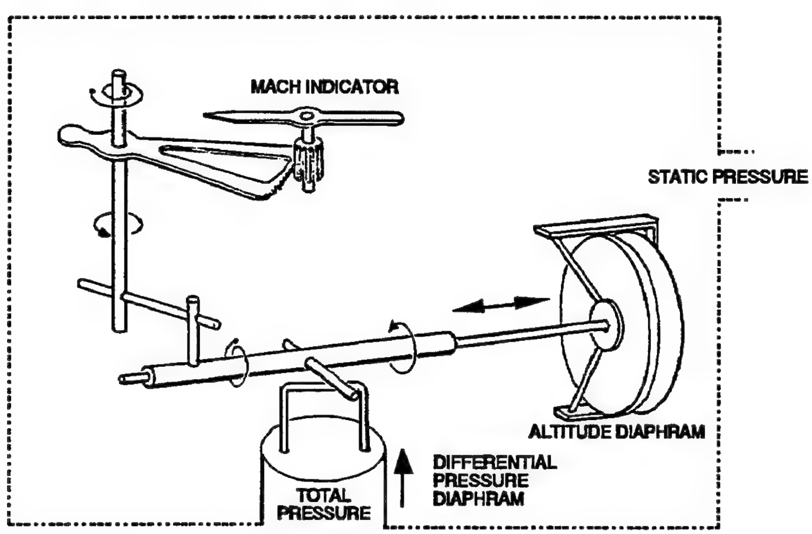


Figure 5.9 Machmeter Schematic

5.5 PITOT-STATICS RELATIONS SUMMARY

The previous two sections have presented the development of the pitot-static relations associated with altitude and airspeed measurement. At this point, a brief review of these relations and their importance to flight testing is in order.

First consider altitude. We found that for the altitudes of interest to aircraft flight testing, geopotential altitude, H, could be considered equal to the geometric altitude, h. Therefore, all the standard atmosphere relations for pressure, density and temperature were derived based on geopotential altitude. Of the three atmospheric parameters, pressure was chosen for use in altitude measurement due to it's insensitivity to temperature inversions. Since aircraft altimeters are designed around the standard atmosphere pressure relations, they will only provide geopotential altitude on a standard day. Therefore, on non-standard or test days, the altitude provided has been defined as pressure altitude, H_c. Thus, independent of day type,

$$H_c = f\,(P_a)$$

The development of the airspeed measurement relations gave basic understanding into how total and static pressure measurements can be used to calculate an aircraft's airspeed. This section also provided insight into why equivalent and calibrated airspeeds were conceived during the development of reliable airspeed indicators for use in aircraft. However, the complexity and number of equations presented may have hampered our understanding of the basic concepts. The key to understanding the airspeed relations for subsonic and supersonic compressible flow lies in the following functional relationships:

Calibrated Airspeed:	$V_c = f\,(q_c)$
Equivalent Airspeed:	$V_e = f\,(q_c, P_a) = f\,(V_c, H_c)$
Mach number:	$M = f\,(q_c, P_a) = f\,(V_c, H_c)$
True Airspeed:	$V_t = f\,(q_c, P_a, \rho_a) = f\,(V_c, H_c, T_a)$

Of importance in flight test is the fact that all of these parameters, except true airspeed, are independent of density or temperature. *As a result, flying a given pressure altitude and calibrated airspeed will provide the same equivalent airspeed and Mach number, independent of the day type.* Additionally, since many aerodynamic effects, particularly in jet engine-airframe performance analysis, are functions of Mach number, this fact plays a major role in flight testing.

5.6 TEMPERATURE MEASUREMENT

Knowledge of the air temperature outside an aircraft in flight is essential to true airspeed determination. Further, accurate temperature measurements are needed for engine control systems, fire control systems, and for accurate bomb release computations.

Typically, temperature measurements on aircraft are performed using a total temperature probe. These devices normally use sensing elements in which the electrical resistance of the element varies with temperature. An electrical bridge balance system is used to show this resistance change on an indicator. Figures 5.10 and 5.11 provide two examples of total temperature probes. While there are many probe designs, the guiding aim has been to reduce or eliminate errors due to conduction, radiation, and angles of attack or sideslip.

To determine freestream ambient temperature from the total temperature measured by the probe, we can use the total temperature equation developed for compressible, subsonic flow (Chapter 4, Reference 1):

$$\frac{T_T}{T_a} = 1 + \frac{\gamma - 1}{2} M^2 \tag{5.60}$$

where

T_T ≡ Freestream Total Temperature
T_a ≡ Freestream Ambient Temperature
M ≡ Freestream Mach number
γ ≡ Ratio of Specific Heats

This relation was derived assuming that the gas is adiabatically slowed at the nose of the total temperature probe between the freestream and the stagnation point. *Since no isentropic assumptions were made, this equation is valid for both subsonic and supersonic flows.*

5.7 AIR DATA SYSTEM ERRORS

Several corrections must be applied to the velocity, airspeed, or temperature values measured by aircraft instrumentation (Reference 3). The indicated altitude and airspeed readings must be corrected for instrument error, pressure lag error, and position error, in that order. Additionally, total temperature readings must be corrected for instrument error and probe heat losses before calculating ambient temperature.

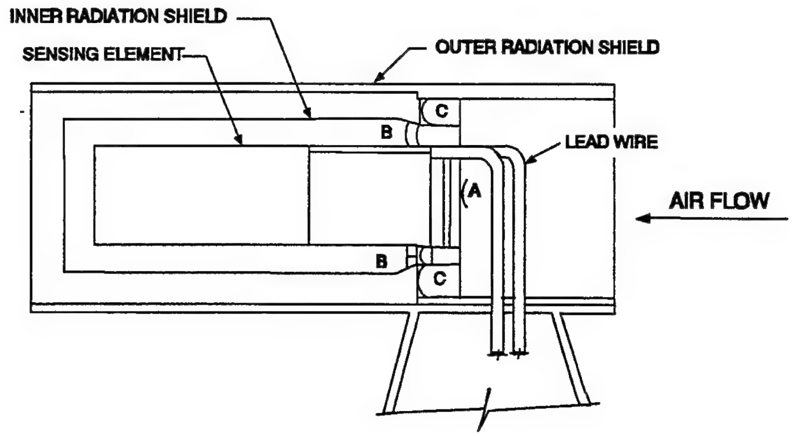


Figure 5.10 Total Temperature Sensor with Boundary Layer Control

Note:
- A. Throat area for flow inside the sensing element.
- B. Throat area for flow outside the sensing element.
- C. Throat area for flow outside the inner shield.

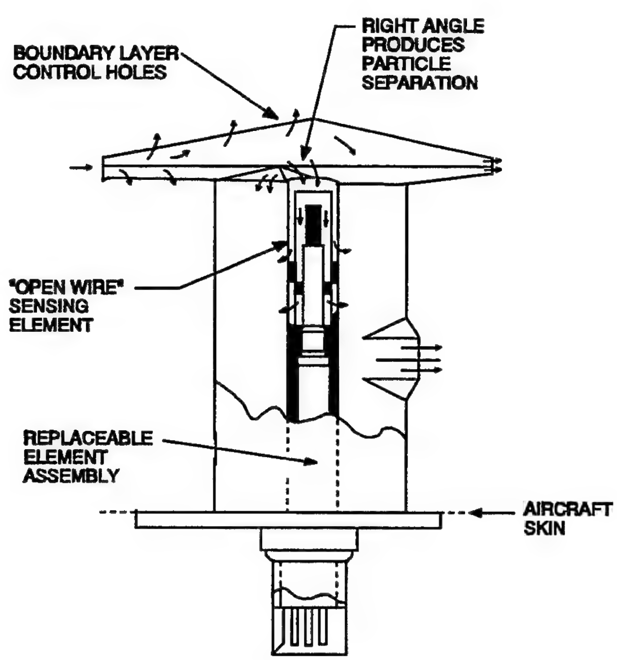


Figure 5.11 Total Temperature Sensor (Non De-iced)

5.7.1 INSTRUMENT ERRORS

The raw values of pressure altitude, calibrated airspeed, Mach number, total temperature, ambient pressure, or differential pressure taken directly from cockpit instruments or other instrumentation are designated as *indicated* values. For example,

H_i ≡ indicated pressure altitude
V_i ≡ indicated airspeed
M_i ≡ indicated Mach number
T_i ≡ indicated total temperature
P_i ≡ indicated ambient pressure
q_{c_i} ≡ indicated differential pressure

Any error in the cockpit instruments or other instrumentation must be removed by the application of instrument calibrations. The instrument error corrections required to correct the indicated values to *instrument corrected* values are defined as

ΔH_{ic} ≡ altitude instrument error correction
ΔV_{ic} ≡ airspeed instrument error correction
ΔM_{ic} ≡ Mach number instrument error correction
ΔT_{ic} ≡ total temperature error correction
ΔP_{ic} ≡ ambient pressure instrument error correction
$\Delta q_{c_{ic}}$ ≡ differential pressure error correction

Thus, applying the error corrections to the indicated values, *assuming no lag error* (Reference section 5.7.2) results in

$$H_{ic} = H_i + \Delta H_{ic} \tag{5.61}$$
$$V_{ic} = V_i + \Delta V_{ic} \tag{5.62}$$
$$M_{ic} = M_i + \Delta M_{ic} \tag{5.63}$$
$$T_{ic} = T_i + \Delta T_{ic} \tag{5.64}$$
$$P_{ic} = P_i + \Delta P_{ic} \tag{5.65}$$
$$q_{c_{ic}} = q_{c_i} + \Delta q_{c_{ic}} \tag{5.66}$$

where

H_{ic} ≡ instrument corrected pressure altitude
V_{ic} ≡ instrument corrected airspeed
M_{ic} ≡ instrument corrected Mach number
T_{ic} ≡ instrument corrected total temperature
P_{ic} ≡ instrument corrected ambient pressure
$q_{c_{ic}}$ ≡ instrument corrected differential pressure

The altimeter, airspeed indicator, Machmeter, and temperature gauge all provide indications according to the equations presented previously. However, it is not possible to perfect an instrument which can represent such nonlinear functions exactly under all flight conditions. The errors in such instrumentation are the result of several factors:

1. Scale error and manufacturing discrepancies. This is primarily due to an imperfect mechanization of the controlling equations.

2. Magnetic Fields. Any change in the relation of metal components changes the associated magnetic fields.

3. Temperature changes. Changes in the temperature can cause expansion and contraction of components affecting the overall reading and possibly increasing or decreasing the friction between moving parts

4. Coulomb and viscous friction. Coulomb friction is simply dry friction, as in the meshing of two gears. Viscous friction is involved between a fluid and solid, for instance, a lubricated bearing.

5. Inertia of moving parts. Inertia is the resistance of any mass to changes in velocity.

The resulting error due to these factors varies with the input value. Additionally, if the input to the instrument is increasing to some fixed final value, the instrument reading will be different than it would have if the input had decreased to that same final value. This effect is called *hysteresis* and must be determined during calibration. An instrument with a large hysteresis must be rejected as it is difficult to account for this effect in flight.

Note that friction and temperature errors are considered tolerances or accuracies of the instrument since they are not dependent on the instrument readings. An instrument with excessive friction or temperature errors should be rejected. However, friction errors can be reduced with the use of a vibrator.

The calibration of cockpit air data instrumentation for instrument error is usually conducted in a laboratory. A known quantity, such as pressure, differential pressure, or resistance (for temperature), as appropriate, is applied to the instrument being tested. The instrument error is determined as the difference between the value calculated from the appropriate equation based on the known input value and the instrument indicated reading. The readings are performed with both increasing and decreasing input in order to determine the instrument hysteresis under static conditions. Figure 12 provides the results of such a calibration for a typical airspeed

indicator. While the hysteresis may be different during a rapidly varying situation, due to the inertia of the moving components and viscous friction, it is not feasible to calibrate instruments for such conditions.

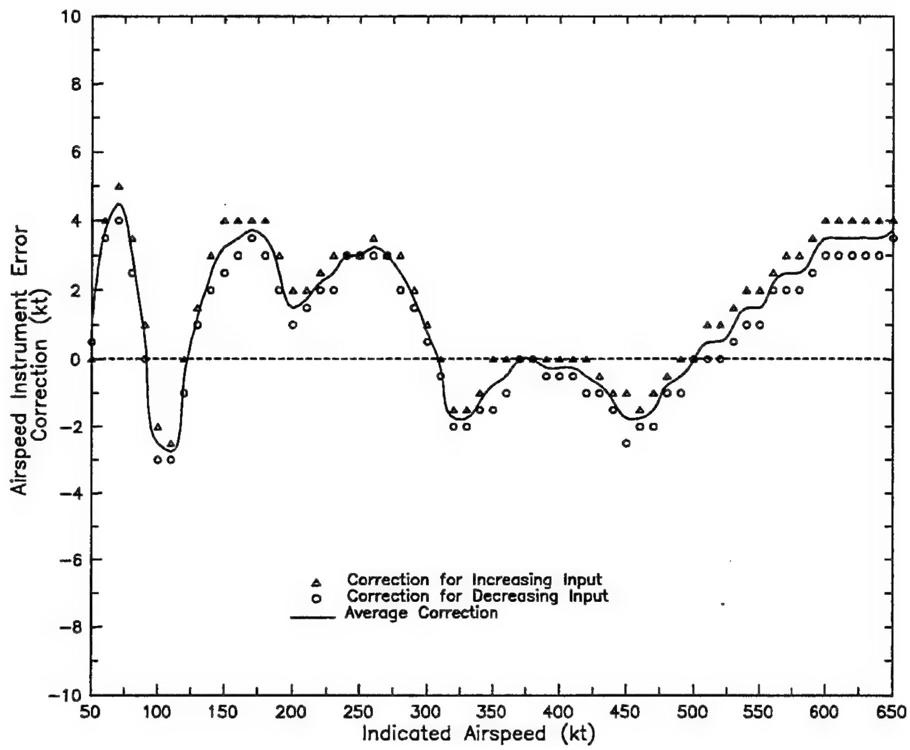


Figure 5.12 Airspeed Indicator Calibration Curve

Newer aircraft are equipped with pressure transducers and temperature sensors which convert measured pressures and resistance to an electrical signal for input to the central air data computer. The air data computer transforms the electrical signal into a pressure or temperature value, as applicable, in engineering units. While this special instrumentation can be calibrated in the laboratory, the overall errors associated with this complete process of converting an input value into a final measurement must be determined by ground testing. These errors can be then mathematically modeled for input into the air data computer or data reduction software.

One final consideration is that as an instrument wears, its calibration changes. Therefore, each instrument should be calibrated periodically. The repeatability of the instrument is determined from the instrument calibration history and must be good for the calibration to be meaningful.

5.7.2 PRESSURE LAG ERROR

Measurements of altitude, Mach number, and indicated airspeed are subject to an error called pressure lag error. This error exists only when the aircraft is changing airspeed or altitude. In this case, there is a time lag between the actual pressure change and when it is indicated on the instrument dial. This effect on the altimeter is obvious; as the aircraft climbs, the instrument will indicate an altitude less than the actual altitude. In the airspeed indicator, the lag may cause a reading too large or too small depending on the proportion of the lag in the total and static pressure systems. Converted to feet or knots, this error is often insignificant. However, it may be significant and should be considered in certain maneuvers such as high speed dives and zoom climbs in which the instrument diaphragms must undergo large pressure rates.

Pressure lag is basically a result of:

1. Pressure drop in the tube due to viscous friction.
2. Inertia of the instrument and air mass in the tubing.
3. Viscous and kinetic friction.
4. Finite speed of pressure propagation, or acoustic lag.

A detailed mathematical treatment of the response of such a system would be difficult and is beyond the scope of this text. It has been found that mathematical prediction of the lag constants has not been satisfactory. A ground determination can be made by placing a known varying pressure source to the total and static pressure ports of the pitot-static tube and then correlating this with the indicated values. Airborne determination is possible, but it is complicated and has not normally resulted in satisfactory performance. For the majority of quasisteady-state performance flight tests, the altimeter and airspeed indicator lag corrections (ΔH_{ic_l} and ΔV_{ic_l}) can be ignored, with some loss in accuracy. A discussion of the lag error calculation can be found in Reference 3 (Chapter 1, Section 4) and Reference 4 (pp 28-47).

5.7.3 POSITION ERROR[3]

Determination of the pressure altitude, calibrated airspeed, and Mach number at which an aircraft is operating is dependent upon the measurement of free stream total pressure, P_T, and free stream static pressure, P_a, by the aircraft pitot-static system.

[3] Based on a similar presentation of the subject in Reference 3, Section 5

Generally, the pressures registered by the pitot-static system differ from free stream pressures as a result of:

1. The existence of other than free stream pressures at the location of pressure sources.

2. Error in the local pressure at the source caused by the pressure sensors themselves.

The resulting error is called position error. In the general case, position error may result from errors at both the static and total pressure sources. Once position error corrections are determined for an aircraft, they can be added to the instrument corrected values of pressure altitude, calibrated airspeed, and Mach number to determine the actual values of these parameters:

$$H_c = H_{ic} + \Delta H_{pc} \tag{5.67}$$

$$V_c = V_{ic} + \Delta V_{pc} \tag{5.68}$$

$$M = M_{ic} + \Delta M_{pc} \tag{5.69}$$

5.7.3.1 TOTAL PRESSURE ERROR

As an aircraft moves through the air, the ambient pressure is disturbed, producing a static pressure field around the aircraft. At subsonic speeds, the flow perturbations due to the aircraft static pressure field are very nearly isentropic in nature and hence do not affect the total pressure. Therefore, as long as the total pressure source is not located behind a propeller, in the wing wake, in a boundary layer, or in a region of localized supersonic flow, the total pressure errors due to the position of the total pressure head are usually negligible. Generally, it is possible to locate the total pressure pickup properly and avoid any difficulty. Aircraft capable of supersonic speeds can be supplied with a noseboom pitot-static system so that the total pressure pickup is located ahead of any shock waves formed by the aircraft. This condition is essential, for it is difficult to correct for total pressure errors which result when oblique shock waves exist ahead of the pickup. The shock wave due to the pickup itself is considered in the calibration equation discussed previously in section 5.4.3. Failure of the total pressure sensor to register the local pressure may result from the shape of the pitot-static head, inclination to the flow angle of attack, α, or angle of sideslip, β, or a combination of all three. Pitot-static tubes have been designed in varied shapes. Some are suitable only for relatively low speeds while others are designed to operate in both subsonic and supersonic flight. If a proper design is selected and the pitot lips are not damaged, there should be no error in total pressure

due to the shape of the probe. Errors in total pressure caused by the angle of incidence of a probe to the relative wind are negligible for most flight conditions. Commonly used probes produce no significant errors at angles of attack or sideslip up to approximately 20°. The quality of any probe can be proven through wind tunnel testing. *Thus, with proper placement, design, and good leak checks of the pitot probe, total pressure error can be assumed to be zero.*

5.7.3.2 STATIC PRESSURE ERROR

The static pressure field surrounding an aircraft in flight is a function of airspeed and altitude as well as the secondary parameters, angle of attack, Mach number, and Reynolds number. Hence, it is seldom possible to find a location for the static pressure source where the free stream pressure is sensed under all flight conditions. An error in the measurement of the static pressure due to the position of the static pressure orifice generally exists.

At subsonic speeds, it is often possible to find some position on the fuselage where the static pressure error is small under all flight conditions. Aircraft limited to subsonic speeds are best instrumented by use of a flush static pressure port in such a position. Examples of such positions are shown in Figure 5.13.

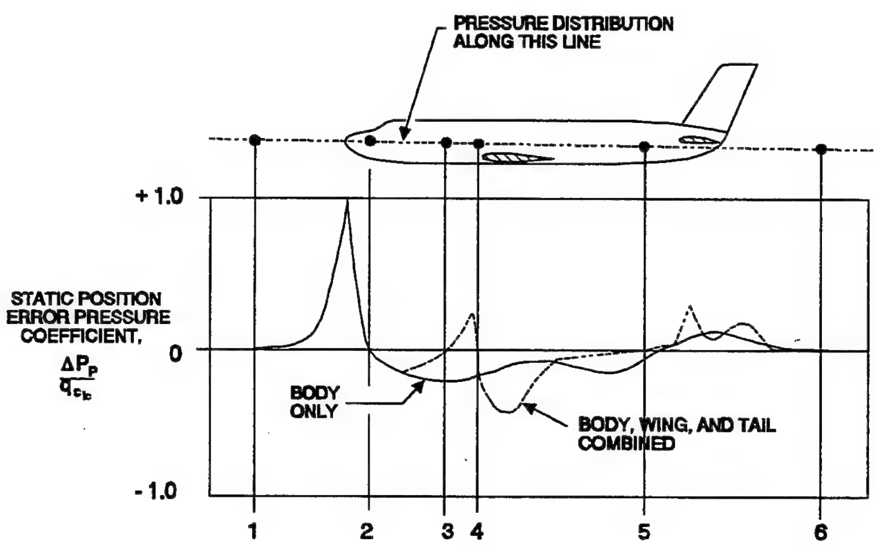


Figure 5.13 Subsonic Static Pressure Distribution

Note: Points 1-6 are locations of minimum static pressure error.

On supersonic aircraft a noseboom installation is advantageous for the measurement of static pressure. At supersonic speeds, when the bow wave is located downstream of the static pressure orifices, there is no error due to the aircraft pressure field. Any error which may exist is a result of the probe itself. Available evidence suggests that free stream static pressure is sensed if the static ports are located more than 8 to 10 tube diameters behind the nose of the pitot-static tube and 4 to 6 diameters in front of the shoulder, as illustrated in Figure 5.14.

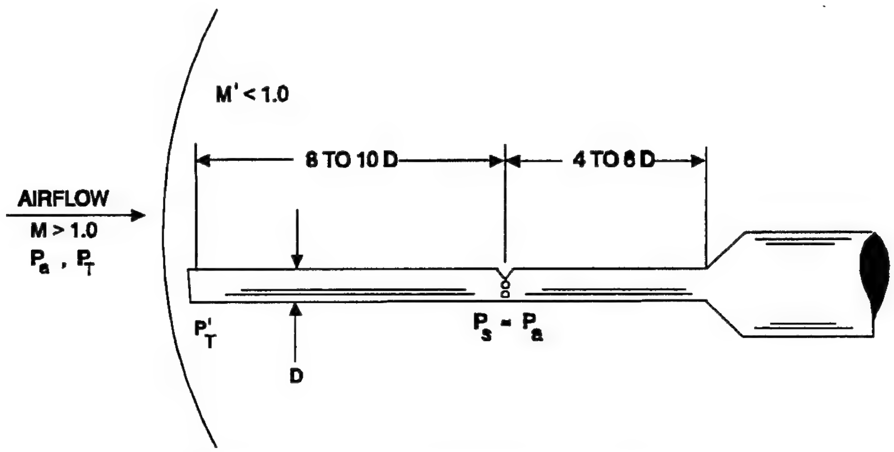


Figure 5.14 Noseboom Schematic

In addition to the static pressure error introduced by the position of the static pressure orifices in the pressure field of the aircraft, there may be error in the registration of the local static pressure due primarily to inclination of flow. Error due to sideslip is often minimized in the case of the flush mounted static ports by manifolding together the static ports located on opposite sides of the fuselage. In the case of boom installations, circumferential location of the static pressure ports reduces the adverse effect of sideslip and angle of attack.

5.7.3.3 STATIC POSITION ERROR

It has been shown that position error is primarily caused at the static pressure source due to the pressure field around the aircraft and the pitot-static probe. The total pressure source, if properly designed, provides negligible error. Thus, we can now develop the relations for position error *based on the assumption that the total pressure error is zero.* The pressure error at the static source, termed *static position error*, is defined as

$$\Delta P_p = P_s - P_a \tag{5.70}$$

where

$P_s \equiv$ static pressure measured at the static pressure source

Since the actual pressure measured at the static source is now static pressure instead of ambient pressure, the resulting measured differential pressure must be redefined as the instrument corrected differential pressure, $q_{c_{ic}}$, given by

$$q_{c_{ic}} = P_T - P_s \tag{5.71}$$

The altimeter relations, equations 5.24 and 5.25, now provide instrument corrected pressure altitude based on the measured value of static pressure. Therefore, for -16,404.2 ft $\leq H_{ic} \leq$ 36,089.24 ft, the static port pressure ratio, δ_s, is defined as:

$$\delta_s \equiv \frac{P_s}{P_{a_{SL}}} = (1 - 6.87558\text{x}10^{-6}\, H_{ic})^{5.2559} \tag{5.72}$$

And for 36,089.24 ft $< H_{ic} \leq$ 65,616.80 ft:

$$\delta_s = 0.223360\; e^{-4.80637x10^{-5}(H_{ic}-36089.24)} \tag{5.73}$$

The subsonic and supersonic instrument corrected Mach number and calibrated airspeed relations must also be rewritten in terms of static pressure and instrument corrected differential pressure, making these parameters instrument corrected Mach number, M_{ic}, and instrument corrected airspeed, V_{ic}. Thus, from equations 34 and 57 we find that at *subsonic speeds*

$$M_{ic} = \left[5\left[\left(\frac{q_{c_{ic}}}{P_s} + 1\right)^{\frac{2}{7}} - 1\right]\right]^{\frac{1}{2}} \tag{5.74}$$

$$V_{ic} = \left[\frac{7\,P_{a_{SL}}}{\rho_{a_{SL}}}\left[\left(\frac{q_{c_{ic}}}{P_{a_{SL}}} + 1\right)^{\frac{2}{7}} - 1\right]\right]^{\frac{1}{2}} \tag{5.75}$$

And from equations 43 and 59, at *supersonic speeds*

$$\frac{q_{c_{ic}}}{P_s} = \frac{166.921\, M_{ic}^7}{\left[7\,M_{ic}^2 - 1\right]^{2.5}} - 1 \tag{5.76}$$

$$\frac{q_{c_{ic}}}{P_{a_{SL}}} = \frac{166.921\left(\frac{V_{ic}}{a_{SL}}\right)^7}{\left[7\left(\frac{V_{ic}}{a_{SL}}\right)^2 - 1\right]^{2.5}} - 1 \tag{5.77}$$

The sign convention used results in the static position error corrections being the same sign as ΔP_p. From the equations presented, it can be seen that if P_s were greater than P_a, the airspeed indicator would indicate a lower than actual value. Therefore, ΔP_p and ΔV_{pc} would both be positive in order to correct V_{ic} to V_c. This same logic applies to ΔH_{pc} and ΔM_{pc}.

Since ΔP_p is the source of position error for all three parameters, mathematical relationships can easily be derived to relate the static position error corrections. The development of these relationships will not be presented, but can be found in section 5.3 of Reference 3. The resulting equations are as follows:

$$\Delta H_{pc} = \frac{\theta_s}{3.61371 \times 10^{-5} /ft} \left(\frac{\Delta P_p}{P_s} \right) \tag{5.78}$$

Subsonic

$$\Delta V_{pc} = \frac{a_{SL}^2 \, \delta_s}{1.4 V_{ic} \left[1 + 0.2 \left(\frac{V_{ic}}{a_{SL}} \right)^2 \right]^{2.5}} \left(\frac{\Delta P_p}{P_s} \right) \tag{5.79}$$

$$\Delta M_{pc} = \frac{(1 + 0.2 \, M_{ic}^2)}{1.4 \, M_{ic}} \left(\frac{\Delta P_p}{P_s} \right) \tag{5.80}$$

Supersonic

$$\Delta V_{pc} = \frac{a_{SL} \, \delta_s}{1168.451} \left(\frac{a_{SL}}{V_{ic}} \right)^6 \frac{\left[7 \left(\frac{V_{ic}}{a_{SL}} \right)^2 - 1 \right]^{3.5}}{\left[2 \left(\frac{V_{ic}}{a_{SL}} \right)^2 - 1 \right]} \left(\frac{\Delta P_p}{P_s} \right) \tag{5.81}$$

$$\Delta M_{pc} = \frac{M_{ic} \left(7 \, M_{ic}^2 - 1 \right)}{7 \, \left(2 \, M_{ic}^2 - 1 \right)} \left(\frac{\Delta P_p}{P_s} \right) \tag{5.82}$$

where θ_s is the static port standard day temperature ratio, given by:

$$\theta_s = 1 - 6.87558 \times 10^{-6} \, H_{ic} \qquad (-16{,}404.2 \text{ ft} \le H_{ic} \le 36{,}089.24 \text{ ft}) \tag{5.83}$$

or

$$\theta_s = 0.751865 \qquad (36{,}089.24 \text{ ft} < H_{ic} \le 65{,}616.80 \text{ ft}) \tag{5.84}$$

and

$$\frac{\Delta P_p}{P_s} \equiv \text{static port position error ratio}$$

These relationships can be found in graphic form in Reference 3 (pp 225 - 261) and are used in data reduction routines to obtain one position error correction from another.

By dimensional analysis, it can be shown that the relationship of static pressure at any point in a pressure field of an aircraft to the free stream ambient pressure depends on Mach number, angle of attack, α, angle of sideslip, β, Reynolds number, R_e, and Prandtl number, P_r.

$$\frac{P_s}{P_a} = f_1\,(M, \alpha, \beta, R_e, P_r)$$

Neglecting heat transfer, the Prandtl number is approximately constant. Additionally, Reynolds number effects are negligible as long as the static source is not located in a thick boundary layer and assuming that sideslip angles are kept small. Therefore, this relation simplifies to

$$\frac{P_s}{P_a} = f_2\,(M, \alpha)$$

With no loss in generality, this equation can be rewritten as

$$\frac{\Delta P_p}{q_{c_{ic}}} = f_3\left(M_{ic}, C_{L_{ic}}\right) \tag{5.85}$$

recalling

$$M_{ic} = f\left(\frac{q_{c_{ic}}}{P_s}\right)$$

and, for a fixed wing geometry,

$$\alpha \to C_{L_{ic}} = \frac{nW}{\frac{1}{2}\gamma P_s M_{ic}^2 S} = \frac{nW}{\delta_s}\,\frac{1}{M_{ic}^2}\,\frac{2}{\gamma S P_{a_{SL}}}$$

where the variable n is defined as the aircraft load factor.

The term $\frac{\Delta P_p}{q_{c_{ic}}}$ is the position error pressure coefficient and is useful in the reduction of position error data. Substituting for $C_{L_{ic}}$ in equation 5.85 to complete our dimensional analysis yields

$$\frac{\Delta P_p}{q_{c_{ic}}} = f_4\left(M_{ic}, \frac{nW}{\delta_s}\right) \tag{5.86}$$

From equation 5.86 it can be seen that there are three major variables in any particular pitot-static system. These are Mach, weight-load factor combination, and the altitude at which the aircraft is flying. It is importance to emphasize one of the assumptions of the derivation of equation 5.86. The angle of sideslip was assumed small. Therefore, we cannot apply this equation at high angles of sideslip.

5.7.3.4 LOW MACH NUMBER STATIC POSITION ERROR

For Mach numbers less than 0.6, the effects of compressibility may be considered negligible. Without introducing serious error, it may be said that the pressure coefficient is a function only of lift coefficient. Since

$$C_L = \frac{2nW}{\rho_{a_{SL}} V_e^2 S}$$

and, in the low Mach range, V_e approximately equals V_{ic}, it can be assumed that

$$C_{L_{ic}} = \frac{2nW}{\rho_{a_{SL}} V_{ic}^2 S}$$

Thus,

$$\frac{\Delta P_p}{q_{c_{ic}}} = f_5\left(\frac{nW}{V_{ic}^2}\right) \tag{5.87}$$

From this equation it can be seen that for low speeds at a constant V_{ic}, the position error, ΔP_p, is a function of nW, or more simply, angle of attack. In addition, from equation 5.79, the velocity position error correction, ΔV_{pc}, is really only a function of V_{ic} and position error ΔP_p. Thus, for aircraft with small weight changes, these functions all generalize into one curve. However, this generalization will not apply to aircraft such as the C-141, where large variations in weight occur. Instead, a family of curves will be generated, as illustrated in Figure 5.15. Notice that, with our assumptions, δ_s was not present in equation 5.87. Therefore, these curves will apply to all altitudes.

From equation 5.78, the altitude position error correction, ΔH_{pc}, is a function of ΔP_P and, noting that

$$\frac{\theta_s}{P_s}\left(\frac{P_{a_{SL}}}{P_{a_{SL}}}\right) = \frac{\theta_s}{\delta_s\, P_{a_{SL}}} = \frac{1}{\sigma_s\, P_{a_{SL}}}$$

the inverse of the density ratio ($1/\sigma_s$). Thus, assuming no weight effects, for any given V_{ic}, ΔH_{pc} is only a function of altitude. As altitude increases, the inverse of the

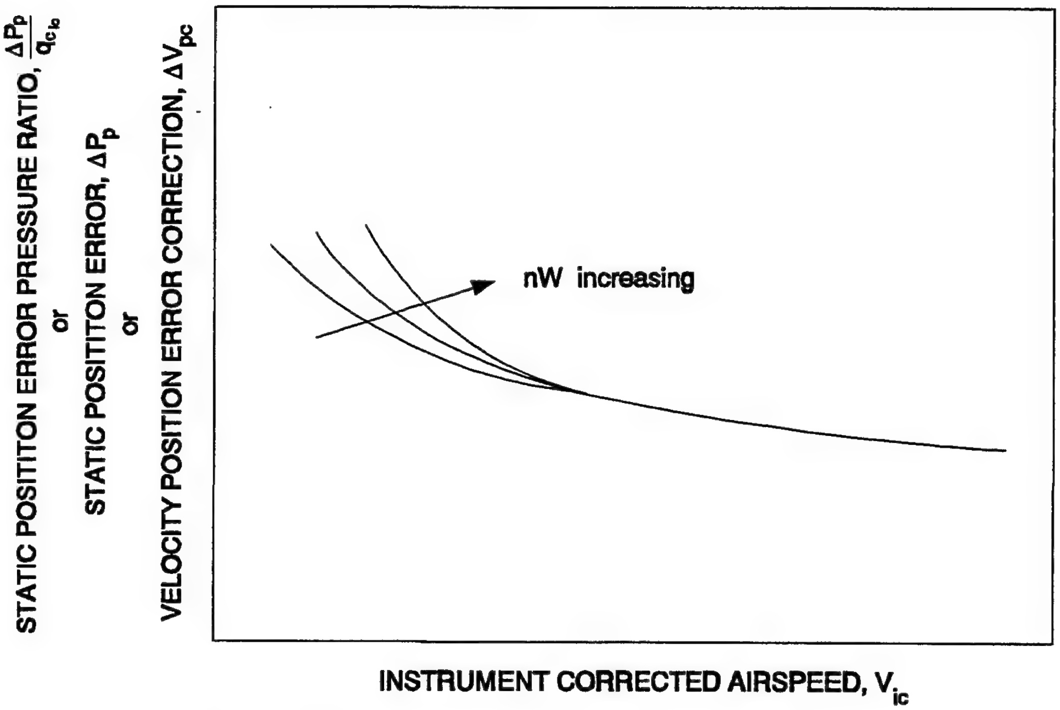


Figure 5.15 Low Speed Static Position Error

density ratio also increases, resulting in an increase in ΔH_{pc} with altitude. This is because a pressure increment is equivalent to a different physical altitude difference at each pressure level in the atmosphere. This results in a series of curves, one for each pressure altitude, as presented in Figure 5.16.

Considering equation 5.80, the Mach number position error correction, ΔM_{pc}, is a function of M_{ic}, ΔP_p, and δ_s. Recall from equation 5.86, $\frac{\Delta P_p}{q_{c_{ic}}}$ (or ΔP_p) was found to be a function of M_{ic} and $\frac{nW}{\delta_s}$ Thus, without the need for the slow speed assumptions, ΔM_{pc} is simply a function of these two parameters. Plotting ΔM_{pc} as a function of M_{ic} results in a family of curves for various $\frac{nW}{\delta_s}$ values, as shown in Figure 5.17. This merely serves to illustrate that for many pitot-static systems in which ΔV_{pc} versus V_{ic} is a single curve, ΔM_{pc} versus M_{ic} is a series of curves, one for each representative altitude, in the low speed regime.

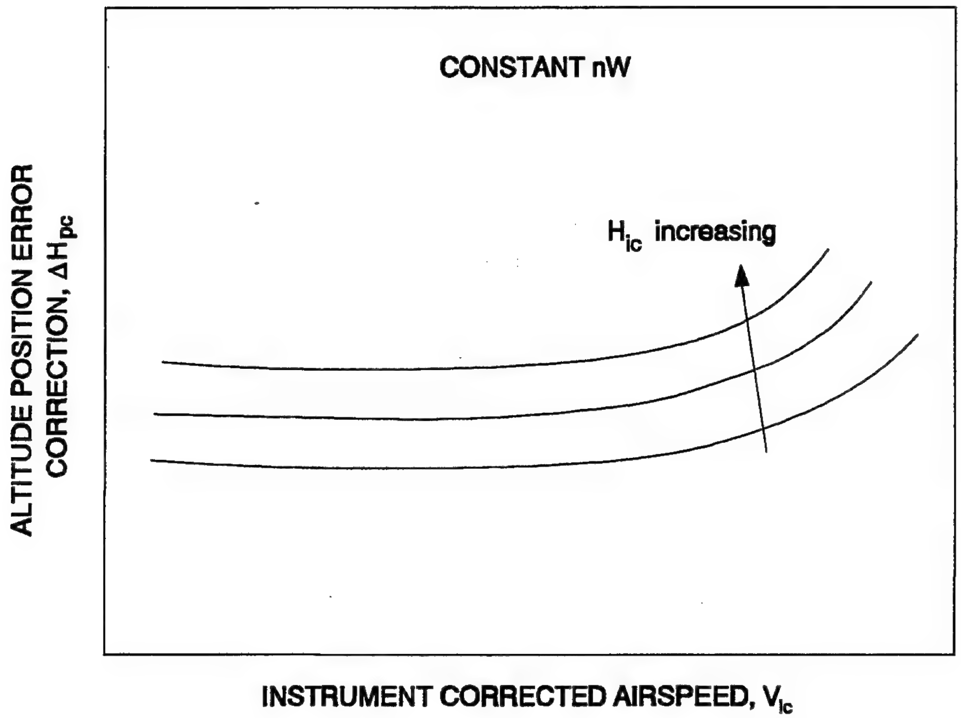


Figure 5.16 Low Speed Altitude Position Error Correction

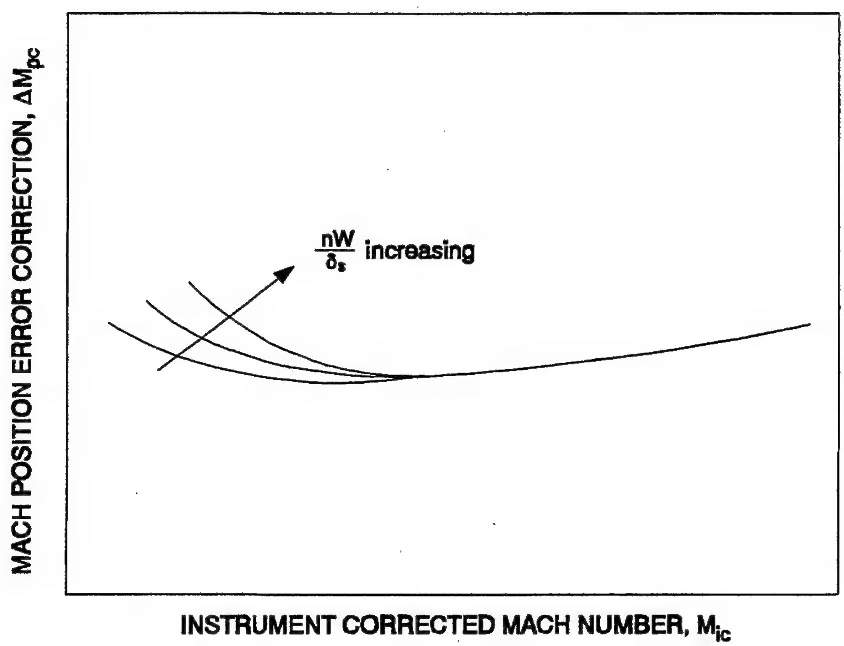


Figure 5.17 Low Speed Mach Position Error Correction

5.7.3.5 MEDIUM SUBSONIC AND TRANSONIC STATIC POSITION ERROR

In the Mach range of 0.6 to 1.1, the position error pressure coefficient will, in general, depend on both M_{ic} and $C_{L_{ic}}$. Thus, equation 5.86 must be considered:

$$\frac{\Delta P_p}{q_{c_{ic}}} = f_4\left(M_{ic}, \frac{nW}{\delta_s}\right)$$

At high speeds, large changes in airspeed produce relatively small changes in $C_{L_{ic}}$. As a result, the effect of lift coefficient diminishes as airspeed increases. Mach number begins to affect the pressure coefficient significantly above 0.5 and becomes the controlling parameter in the high speed regime. The existence of any $C_{L_{ic}}$ effect should be investigated by plotting curves of $\frac{\Delta P_p}{q_{c_{ic}}}$ versus M_{ic} for constant values of $\frac{nW}{\delta_s}$. The result of a typical noseboom system is shown in Figure 5.18. This curve would be a single curve if there were no $C_{L_{ic}}$ effects.

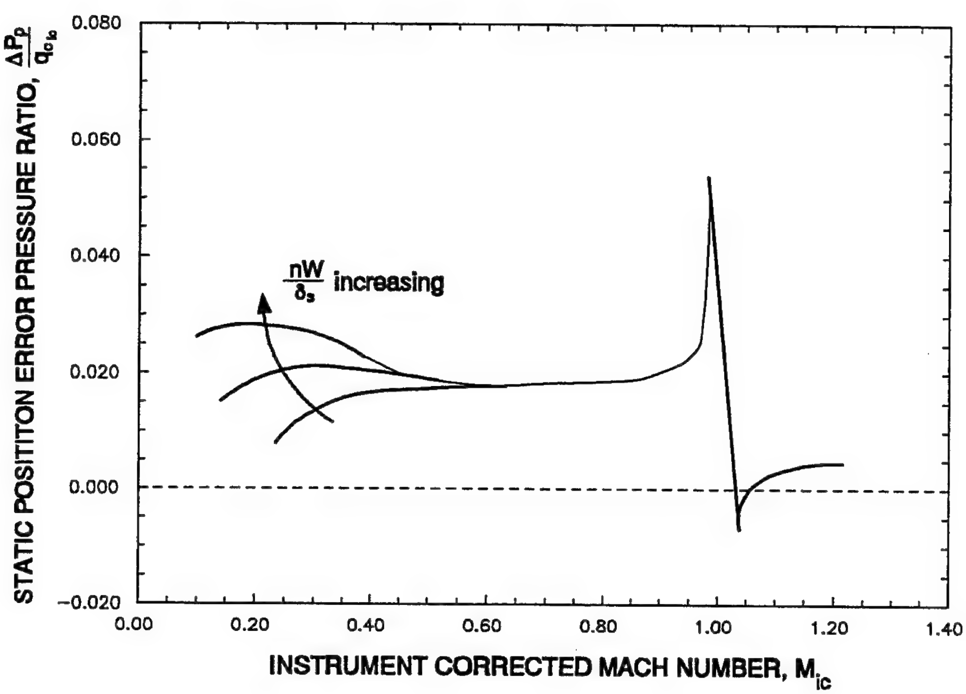


Figure 5.18 Static Position Error Pressure Coefficient

As the aircraft passes critical Mach, shock formation occurs over the various components of the aircraft. As these shock waves approach the static source, the sharp pressure rise which precedes the shock wave will drastically affect the static system.

In this region, $\frac{\Delta P_p}{q_{c_{ic}}}$ rises rapidly and is strictly a function of Mach number.

Again considering equation 5.79, we find that ΔV_{pc} is a function of δ_s, $1/V_{ic}$, and $\frac{\Delta P_p}{q_{c_{ic}}}$ (or M_{ic}). From equations 5.74 - 5.77, for a particular value of M_{ic}, V_{ic} decreases as altitude increases. Thus, for a given load factor and weight combination, plotting ΔV_{pc} as a function of V_{ic} results in a family of curves which shift horizontally with altitude, as illustrated in Figure 5.19. In contrast, plotting against M_{ic} will result in a vertical shift with altitude.

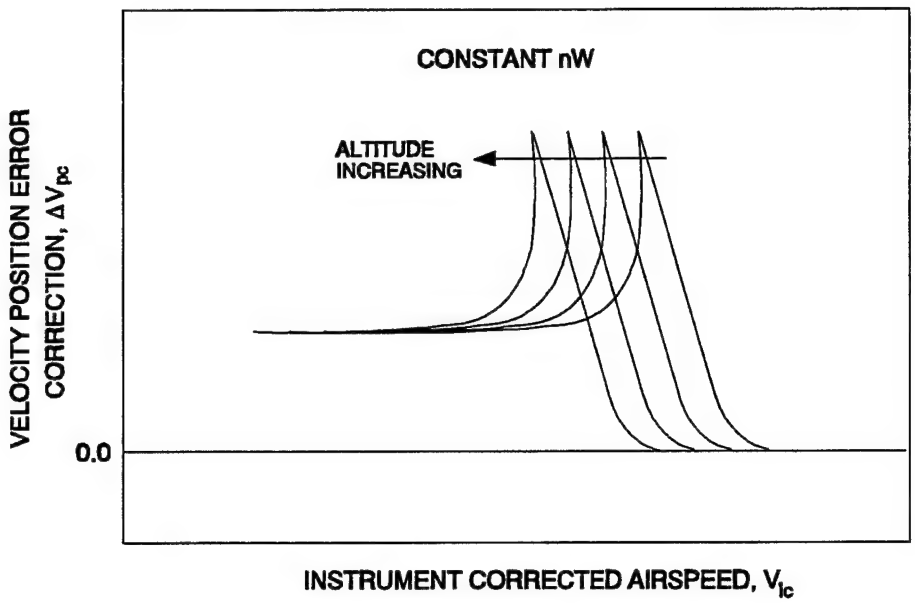


Figure 5.19 High Subsonic and Transonic Velocity Position Error Correction

Recall from equation 5.78, ΔH_{pc} is a function of θ_s and $\frac{\Delta P_p}{q_{c_{ic}}}$. In the slow speed case we found that when plotted against V_{ic}, the lines of constant altitude shifted vertically, as seen previously in Figure 5.16. However, for the medium subsonic and transonic region, this same effect is only seen when ΔH_{pc} is plotted against M_{ic} due to the θ_s term. If plotted against V_{ic}, the lines are displaced both horizontally and vertically, the horizontal displacement produced for the same reasons as discussed for ΔV_{pc}.

5.7.3.6 SUPERSONIC MACH EFFECTS

An aircraft capable of supersonic flight should be equipped with a noseboom installation. In this case, the aircraft bow wave passes behind the static pressure ports at a M_{ic} of approximately 1.03. At higher Mach, the effect of lift coefficient on the position error pressure coefficient is zero, as the pressure field of the aircraft is not felt in front of the bow wave. Therefore, any pressure error that does exist is a function of Mach number only. Usually, this error is quite small and assumed to be negligible.

5.7.3.7 EXTRAPOLATION OF RESULTS

The position error pressure coefficient has been shown to depend on both M_{ic} and $C_{L_{ic}}$.

$$\frac{\Delta P_p}{q_{c_{ic}}} = f\,(M_{ic},\ C_{L_{ic}}) \text{ or } f\left(M_{ic},\ \frac{nW}{\delta_s}\right)$$

For aircraft with small weight effects and all aircraft at higher speeds, the effect of lift coefficient (angle of attack) variation is negligible. In these cases, a calibration at one altitude can be extrapolated to other altitudes at the same Mach, since

$$\frac{\Delta P_p}{q_{c_{ic}}} = f(M_{ic})$$

However, the existence of $C_{L_{ic}}$ effects should be investigated for a particular aircraft before applying this approximation. This can be done by first performing tests at at least two altitudes and at heavy and light weights; and then plotting the curves of $\frac{\Delta P_p}{q_{c_{ic}}}$ as a function of M_{ic} for constant values of $\frac{nW}{\delta_s}$. A single line will occur if no weight effects are present.

Now, let us begin the derivation of the equation which will, under our assumption of negligible lift coefficient effects, allow us to extrapolate static position error to other altitudes. First, equation 5.74 can be rearranged to provide the following expression for P_T/P_s as a function of M_{ic}.

$$\frac{P_T}{P_s} = \left(1 + 0.2\,M_{ic}^2\right)^{3.5}$$

Note that in using this equation, we have assumed the total pressure measurement had no error. Therefore, we can differentiate this equation with respect to M_{ic} at constant P_T; and then substitute for P_T in the resulting expression to provide the following differential equation:

$$\frac{dP_s}{dM_{ic}} = \frac{-1.4\,P_s\,M_{ic}}{1 + 0.2\,M_{ic}^2}$$

If we make the approximations:

$$dP_s \cong P_s - P_a = \Delta P_P$$

and

$$dM_{ic} \cong M_{ic} - M = -\Delta M_{pc}$$

and then substitute them into the previous differential equation, we find

$$\frac{\Delta P_p}{\Delta M_{pc}} \approx \frac{1.4\, P_s\, M_{ic}}{1 + 0.2\, M_{ic}^2} \tag{5.88}$$

Substituting the following approximations into equation 5.7;

$$dP \cong P_s - P_a = \Delta P_p$$

and

$$dH \cong H_c - H_{ic} = \Delta H_{pc}$$

results in

$$\frac{\Delta P_p}{P_s} \approx -\frac{g_{SL}}{R T_a} \Delta H_{pc}$$

where T_a is the test day ambient temperature. Combining this result with equation 5.88 and substituting for the gravitational acceleration and dry air constants produces

$$\frac{\Delta M_{pc}}{\Delta H_{pc}} = 0.007438 \frac{K}{ft} \frac{(1 + 0.2\, M_{ic}^2)}{T_a M_{ic}} \tag{5.89}$$

Now, *if we extrapolate from altitude 1 to altitude 2, for the same instrument corrected Mach number and same Mach position error correction (i.e. the same Mach number),*

$$\Delta H_{pc_2} = \Delta H_{pc_1} \frac{T_{a_2}}{T_{a_1}} \tag{5.90}$$

where T_{a_1} and T_{a_2} are the test day ambient temperatures corresponding to H_{c_1} and H_{c_2}, respectively. However, since we do not know T_{a_2}, we can replace the ratio of test day temperatures with the ratio of corresponding standard day temperatures. This equation can then be used for ΔH_{pc} errors up to 3000 feet with negligible loss of accuracy.

5.7.3.8 PRESENTATION OF POSITION ERROR CORRECTIONS

As discussed earlier, position error is expected to be most dependent upon Mach number, configuration, loading, and angle of attack or $\frac{nW}{\delta}$. The instrument corrected form of Mach number, M_{ic}, and pressure ratio, δ_s, are generally used as the independent variables. The equations use these values since, in most data reduction problems, the instrument corrected values are known, while the calibrated values are unknown. We could also evaluate position error as a function of calibrated values

instead of instrument corrected values.

Most test pitot-static nosebooms are very insensitive to changes in angle of attack or sideslip. In fact, angles of 10° or more may have little effect on the static or total pressure readings. Insensitivity to angle of attack simplifies the calibration problem because data taken at one altitude can be extrapolated to other altitudes, and the number of calibration flights can be reduced. The sensitivity to angle of attack should be determined early for a particular configuration and loading. This is done by comparing measurements made at constant M_{ic} at different $\frac{nW}{\delta_s}$ test points. Tests at a different weight, altitude, or load factor will suffice; but testing at different altitudes is generally easier to perform and will affect $\frac{nW}{\delta_s}$ the most. Keep in mind that the Reynolds number effect may also become significant with large changes in altitude.

When comparing the results of testing, either of two plots may be used: $\frac{\Delta P_p}{q_{c_{ic}}}$ as a function of M_{ic}, or ΔM_{pc} as a function of M_{ic}. As shown in Figure 5.20, when there is no sensitivity to angle of attack, the results of tests at different $\frac{nW}{\delta_s}$ values converge. At low Mach, $\frac{nW}{\delta_s}$ breakoffs will usually occur for any pitot-static installation. Since most performance evaluations such as climb, cruise, and descent are done between 0.6 and 0.95 Mach, the break offs are of no consequence. Therefore, the results can be extrapolated to other altitudes and weights. Conversely, if the results do not converge, position error cannot be extrapolated, except over small changes in altitude, weight, and loading. Thus, separate measurements must be made throughout the operating $\frac{nW}{\delta_s}$ range of the aircraft.

There are numerous methods of presenting final position error data in graphic form for a given aircraft configuration and loading. Generally, when the total pressure error is insignificant, the primary graph is that of $\frac{\Delta P_p}{q_{c_{ic}}}$ as a function of M_{ic} at various $\frac{nW}{\delta_s}$. This plot should be presented with data points and curve fits of this data fared in. The fared values can be used for extrapolation or computation of other forms of position error. Some form of regression may be used to curve-fit the data, but the curves cannot be extrapolated outside the data interval. Other typical graphs include ΔV_{pc} as a function of V_{ic}, ΔH_{pc} as a function of V_{ic}, and ΔM_{pc} as a function of M_{ic}, all at various $\frac{nW}{\delta_s}$, as appropriate.

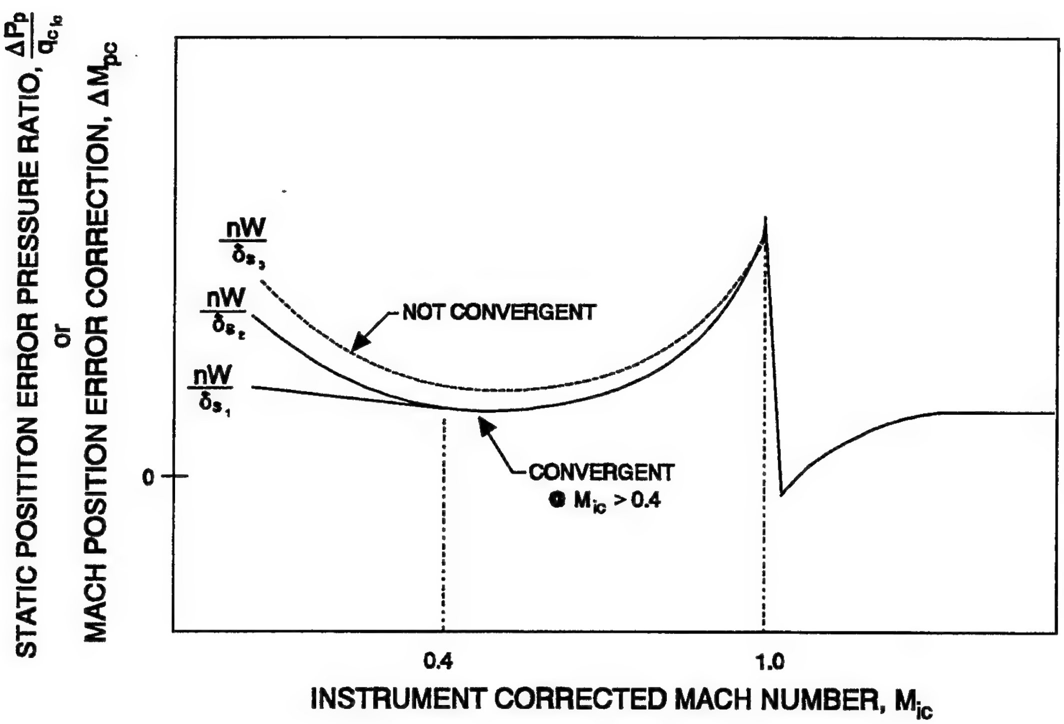


Figure 5.20 Static Position Error Pressure Coefficient Convergence Comparison

5.7.4 TOTAL TEMPERATURE PROBE ERROR

In reality, the air flow is not perfectly adiabatically slowed at the nose of the total temperature probe, making the total temperature measurement in error. Thus, a recovery factor, K_t, must be added to modify the kinetic term of equation 5.60 before using this equation to determine the freestream ambient temperature, resulting in

$$\frac{T_{ic}}{T_a} = 1 + \frac{\gamma - 1}{2} K_t M^2 \tag{5.91}$$

Notice the total freestream temperature, T_T, has been replaced with the total temperature measured by the probe, T_{ic}, defined as:

$T_{ic} \equiv$ instrument corrected Total Temperature

Substituting for $\gamma = 1.4$, this relation simplifies to

$$\frac{T_{ic}}{T_a} = 1 + \frac{K_t}{5} M^2 \tag{5.92}$$

The parameter K_t is most often used to indicate how closely the total temperature sensor actually observes the actual total temperature. This parameter, in effect, provides a measure of how adiabatic the temperature recovery process is (is heat added or lost?). A value of 1.0 for K_t is ideal, but values greater than 1.0 may be

observed when heat is added to the sensors by conduction (hot material around the sensor) or radiation (exposure to direct sunlight). The test conditions must be selected to minimize this type of interference. In general, the value of K_t varies from 0.7 to 1.0. However, for flight test probes, a range of 0.95 to 1.0 is more common.

There are a number of errors in a temperature indicating system which can effect the recovery factor. These may, in certain installations, cause the recovery to vary with airspeed, but in the general case the recovery factor is a constant value. The following include the more significant error sources to consider:

1. *Resistance - Temperature Calibration.* In general, it is not possible to build a resistance temperature sensing element which exactly matches the prescribed resistance - temperature curve. A full calibration of each probe must be made, and the correction must be applied to the data.

2. *Conduction Error.* It is difficult to make a clear separation between recovery errors and errors caused by heat flow from the temperature sensing element to the surrounding structure. This error can be reduced by insulating the probe.

3. *Radiation Error.* When the total temperature being measured is relatively high, heat is radiated from the sensing element, resulting in a reduced indication of temperature. This effect is increased at very high altitude. Radiation error is usually negligible for well-designed sensors when the Mach is less than 3.0 and the altitude below 40,000 feet.

4. *Time Constant.* The time constant is defined as the time required for a certain percentage of the response to an instantaneous change in temperature to be indicated on the instrument. When the temperature is not changing or is changing at an extremely slow rate, the time constant introduces no error. Practical application of a time constant in flight is extremely difficult because the rate of change in temperature with respect to time must be known. The practical solution is to use steady state testing.

Normally temperature probe calibration can be done simultaneously with pitot-static calibration. Indicated total temperature, total temperature instrument corrections, aircraft Mach number, and an accurate ambient temperature are the necessary data. The ambient temperature may be obtained from a pacer aircraft, weather balloon, or calibrated thermometer. Accurate ambient temperature may be particularly difficult to obtain on a tower flyby test because of steep temperature gradients near the Earth's surface and low sun angle early in the morning. Although turbulent air provides mixing and a better sample of ambient temperature to the tower thermometer, turbulence is bad for position error data collection.

The temperature probe recovery factor at a given Mach number and ambient temperature can be found using equation 5.92. Solving for K_t,

$$K_t = \left(\frac{T_{ic}}{T_a} - 1\right)\left(\frac{5}{M^2}\right) \tag{5.93}$$

The results for subsonic calibrations can be plotted as the temperature parameter, $\left(\frac{T_{ic}}{T_a} - 1\right)$, as a function of the Mach parameter, $M^2/5$. These data should fall on a straight line that, when extrapolated, intersects the origin. However, if the curve is offset up or down, a bias, ΔT_B, is present, most likely due to instrument errors not accounted for in the instrument calibration. The slope of this line represents K_t, as illustrated in Figure 5.21.

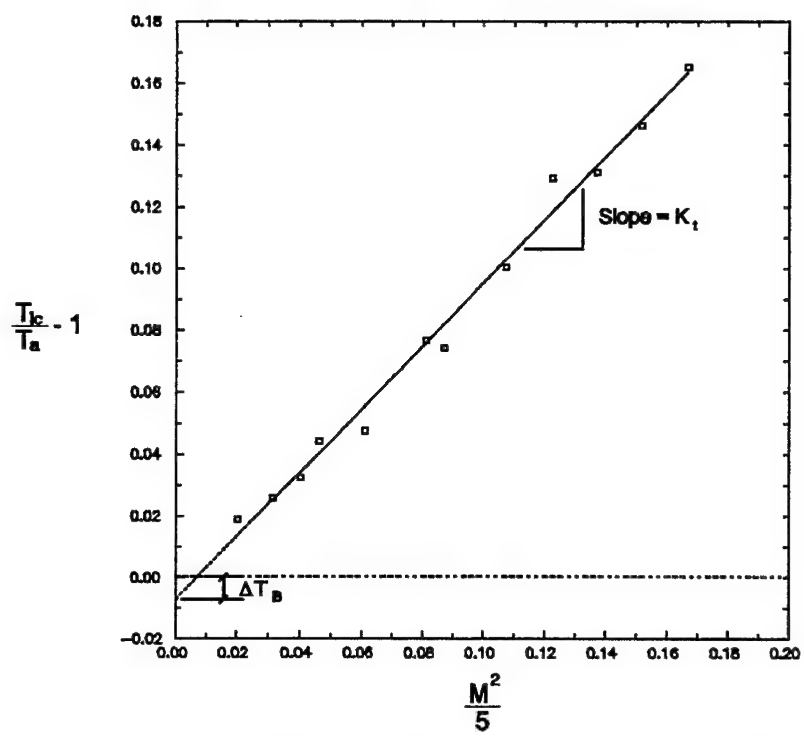


Figure 5.21 Example Temperature Probe Recovery Factor Analysis

5.8 PITOT-STATIC SYSTEM TYPES

Many different types of pitot-static systems exist. These systems are classified primarily based upon the mounting location, as discussed below.

5.8.1 FUSELAGE MOUNTED SYSTEMS

Due to the curvature of the nose section, a low pressure area is created which tends to become lower with increasing airspeed or decreasing C_L. The static source can be placed where the position error is small for the important phases of flight such as takeoff, landing, or cruise, and will remain relatively small for the entire performance envelope if the aircraft operates below the transonic range. This type of system is subject to sideslip errors. By cross manifolding static sources located on opposite sides of the aircraft nose, the error can be minimized; however, the error will still be present and noticeable. The system is extremely poor for transonic and supersonic operation. Because so many factors can affect the formation of shock waves on the fuselage, the error becomes erratic, non-repeatable, and assumes gigantic proportions in the high supersonic regime.

5.8.2 NOSEBOOM SYSTEMS

In noseboom systems, the static source lies in a high pressure area forward of the fuselage. The pressure tends to increase further with increasing airspeed up to 1.0 Mach number. As seen in Figure 5.20, the position error tends to generalize into a single curve below the critical Mach number. Aircraft configuration changes, angles of attack, and sideslip up to approximately 15° do not noticeably affect the system. The various position errors, while large at high subsonic speeds, are consistent and repeatable throughout the flight envelope. The position errors drop to near zero above Mach 1.0 and remain constant out to Mach numbers of approximately 2.0 to 2.5.

Because of its major advantages, the noseboom system is used on nearly all test aircraft throughout performance and flying qualities testing. It is the only system which can meet the accuracy and repeatability requirements of flight test work.

5.8.3 WINGBOOM SYSTEMS

A wingboom system has basically the same advantages as a noseboom system until shock waves are formed in the transonic area. The shock waves from the fuselage then impinge upon the static sources. This results in erratic and inconsistent errors above that speed into the supersonic range.

5.8.4 COMPENSATED SYSTEMS

Pitot-static heads have been designed to create a local low pressure area in the vicinity of the static sources. Such a head can be milled to compensate almost exactly for the general high pressure region existing in the area of a nose mounted test boom. Thus, position error below critical Mach can be brought almost to zero. However, there are several disadvantages to such a system. In supersonic flight, where free stream conditions exist around the boom, the curved head creates a large built-in error which increases with increasing Mach number. Further, the error does not generalize, but tends to exhibit altitude breakoffs at both subsonic and supersonic speeds. For these reasons, the system is normally unsuitable for flight test purposes and its operational use is below 2.0 Mach number.

5.9 AIR DATA SYSTEM CALIBRATION FLIGHT TESTING

The initial step for any test point flown is to measure the pressure and density of the atmosphere and the velocity of the test vehicle to allow proper correlation and standardization of test data. There are restrictions in the current state-of-the-art as to what properties can be accurately measured. For example, density cannot be determined from a direct reading instrument. With few exceptions, test aircraft are generally equipped with pitot-static systems and total temperature probes to provide measurements of ambient pressure, total pressure, and total temperature; all of which are required to calculate the atmospheric properties and aircraft velocity. However, there are inaccuracies within each measuring system which must be determined and corrected for. This is the reason for air data system calibration flight testing

It is obvious that air data system calibration testing is required to insure accurate information is provided to the pilot, allowing safe operation of the aircraft throughout all phases of flight. However, the importance of this testing as a prerequisite to other testing, such as performance, flying qualities, or weapons delivery testing, should not be underestimated. Failure to properly correct for pitot-static and temperature errors will render worthless all data collected. For this reason, calibration tests of the pitot-static and temperature systems should be accomplished early in any test program.

The objective of air data system calibration testing is to determine the test aircraft pitot-static system position error corrections throughout the flight envelope, as well as the total temperature probe recovery factor. With this calibration information, key air data parameters, such as pressure altitude, H_c, calibrated airspeed, V_c, and ambient temperature, T_a, can be accurately determined. Guidance for testing of pitot-static systems can be found in MIL-P-26292C, Pitot and Static Pressure Systems, Installation and Inspection of (Reference 5). Extracts from this military specification are found in Section 10.

As discussed previously, for a given configuration and loading, position error is most sensitive to Mach number, and angle of attack, depending upon the type of static source. Current flight test methods available for determining static position error corrections can be grouped into two categories: altitude comparison methods and airspeed comparison methods. The choice of test method is driven by the altitude and airspeed range to be calibrated, as well as the accuracy of available "truth sources". For example, most altitude comparison methods can provide pressure altitude within

± 20 - 50 ft, while the airspeed comparison methods can provide ground speed within ± 2 - 4 kt. For the Edwards runway (approx 2,300 ft PA), if flying at 400 kt, an altitude position error correction of 50 feet corresponds to approximately one knot of correction in airspeed and a three thousandths correction to Mach number. Thus, an altitude comparison method, such as ground flyby, would yield the most accurate results. But this same 50 ft altimeter position error correction while flying at only 100 kt corresponds to approximately five knots of correction in airspeed and a one hundredth correction in Mach number, making an airspeed comparison method, such as ground speed course, the most accurate. Additionally, using both an altitude and airspeed comparison method in a flight regime where both methods yield the same accuracy of results is crucial to the validation of the zero total pressure error assumption. Only when this error has been found to be negligible may the equations of section 5.7.3.3 be used to determine airspeed and Mach number position error corrections based on measured altitude position error corrections, or altitude and airspeed position error corrections based on measured Mach number position error corrections.

Several classical flight test techniques for calibrating aircraft pitot-static systems are presented. These techniques include altitude and airspeed comparisons that can be categorized into three groups: low altitude subsonic, all altitude subsonic, and all altitude subsonic-supersonic. The low altitude subsonic techniques include the altitude comparison using ground flyby (flyby) and the airspeed comparison using ground speed course (ground speed course) methods. The flyby method provides altitude calibration while the ground speed course method provides Mach number calibration. Used together, these two methods can provide a determination of total and static pressure error for the test aircraft at a single altitude near the local ground level. The all altitude subsonic methods include the pacer and altitude comparison using the trailing bomb/cone (trailing cone) methods. The pacer method provides both airspeed and altitude calibrations while the trailing cone method provides altitude calibrations only. Finally, the all altitude subsonic-supersonic techniques include the altitude comparison using pressure/temperature survey (survey), altitude comparison using smoke-laying aircraft (smoke trail), and the airspeed comparison using all altitude speed course (all altitude speed course) methods. The two altitude comparison flight test techniques can be used to obtain an altitude calibration throughout the speed range of the aircraft (subsonic to supersonic). They are primarily used to obtain transonic and supersonic calibrations to supplement the subsonic data obtained from the pace or trailing cone methods. The all altitude speed

course method provides Mach number calibration based on inertial airspeed measurements. This method, used in combination with one of the two altitude comparison methods, will allow determination of both the static and total pressure errors at any altitude and airspeed.

Current pitot-static systems are limited in their ability to accurately measure air data at very high angles of attack and at airspeeds above 3.5 Mach number. With the current and upcoming testing of high angle of attack and hypersonic aircraft, the need for better air data measurement instrumentation has arisen. Recently, research has been conducted using optical air data techniques. The results of flight testing these methods are discussed in Reference 6.

There are a few important tips on flying technique during pitot-static calibration flights. During stabilized points, the aircraft should be coordinated, and altitude and angle of attack should be held as steady as possible. Pitch bobbling or sideslip may induce error, so resist making a last-second correction. A slight climb or descent may cause the pilot to read the wrong altitude, particularly if there is any delay in reading the instrument. If altimeter position error is being evaluated, read the altimeter first. A slight error in the airspeed reading will not have much effect. In addition, make sure all ground blocks and in-flight data are read with 29.92 set in the Kollsman window of the altimeter.

5.9.1 ALTITUDE COMPARISON USING GROUND FLYBY FLIGHT TEST TECHNIQUE

5.9.1.1 TEST METHOD

The altitude comparison using the ground flyby (flyby) flight test technique produces a fairly accurate calibrated altitude, H_c, by triangulation, as illustrated in Figure 22. (Reference 3, Section 5.6.1) A designated facility, generally a tower, is equipped with a theodolite to measure the aircraft geometric altitude above a flyby line painted along the desired ground track of the test aircraft. The theodolite eyepiece is a known distance from a measurement grid and a flyby line marking the aircraft ground track. Additionally, this facility is equipped with a calibrated altimeter and temperature sensor for measuring pressure altitude and ambient temperature. The test aircraft is flown precisely above the flyby line past the test facility at constant altitude and airspeed. Upon each pass, the test aircraft is sighted through the theodolite and the theodolite reading is recorded along with current pressure altitude and ambient temperature at the level of the eyepiece peep sight. The Theodolite reading represents the grid distance from the zero line to the location of the aircraft pressure transducer or altimeter. Test aircraft passes should be made as low as possible to allow ground test engineers to obtain accurate altitude measurements, but no lower than one full wing span above the ground to keep the aircraft out of ground effect. Required test aircraft data to be recorded upon each pass include: altitude, airspeed, fuel, time, and if available, total temperature.

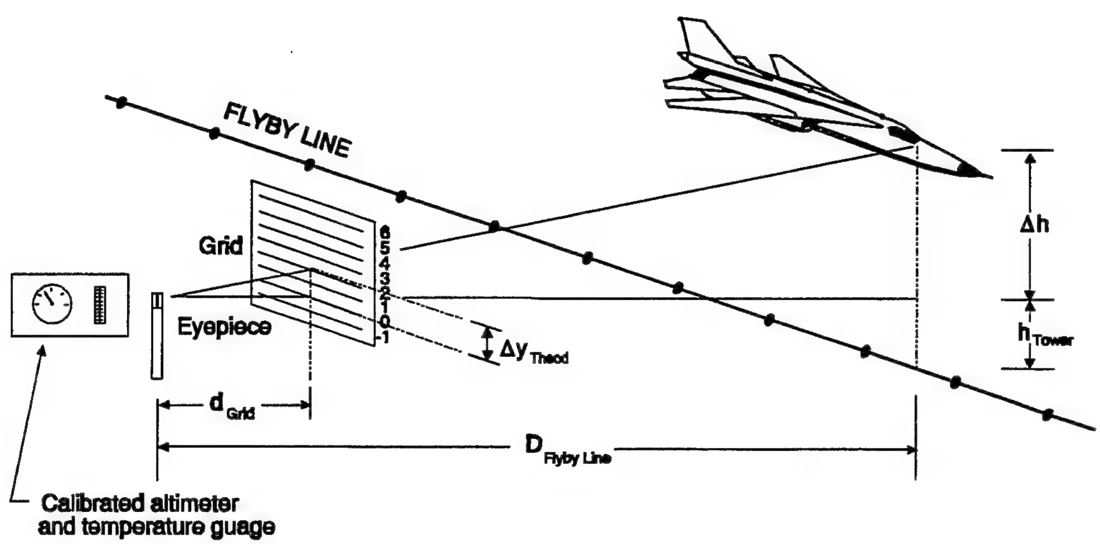


Figure 5.22 Ground Flyby using Theodolite Schematic

Although the flyby method is simple, accurate, and requires no sophisticated equipment, it has some disadvantages: it does not provide a calibration of airspeed; is limited to subsonic flight. Additionally, angle of attack, or $\frac{nW}{\delta}$, effects due to changing aircraft weight as fuel is burned are most prevalent at low speeds. Thus, all low speed points should be flown at a variety of gross weights to determine position error corrections over the range of aircraft weight, if possible. Note that the that the altitude error can also be determined using geometric altitude measurements from a radar or global positioning system (GPS) instead of the theodolite.

5.9.1.2 DATA ANALYSIS

5.9.1.2.1 Test Aircraft Pressure Altitude

The calibrated altitude of the test aircraft, H_c, is the sum of the pressure altitude of the theodolite at the time the point was flown, $H_{c_{Tower}}$, and the *pressure altitude* distance above the theodolite grid zero line, $\Delta H_{c_{Theod}}$. Thus,

$$H_c = H_{c_{Tower}} + \Delta H_{c_{Theod}} \tag{5.94}$$

Using Figure 5.22, we can apply geometric methods to determine the test aircraft *physical* distance above the theodolite grid zero line, Δh. Since the ratios of the sides of two similar triangles are the same, we find:

$$\frac{\Delta y_{Theod}}{d} = \frac{\Delta h}{D}$$

Solving for Δh,

$$\Delta h = \frac{\Delta y_{Theod}\, D}{d} = \Delta y_{Theod}\, K_{Theod} \tag{5.95}$$

where

Δy_{Theod} ≡ theodolite reading

and

K_{Theod} ≡ theodolite reading conversion factor

$$= \frac{D}{d}$$

At Edwards AFB, there are currently two facilities available to use for tower fly-by testing: The Fly-by Tower, with a conversion factor of 31.4 ft/in, and the East Askania Tower, with a conversion factor of 27.1 ft/in.

We can use equation 5.28 to convert Δh into the pressure altitude distance required in equation 5.94. This results in

$$\Delta H_{c_{Theod}} = \Delta h \left(\frac{T_{a_{SD}}}{T_a}\right)_{TWR} = \Delta y_{Theod}\, K_{Theod} \left(\frac{T_{a_{SD}}}{T_a}\right)_{TWR} \tag{5.96}$$

where

$T_{a_{SD}}$ ≡ tower standard day temperature

T_a ≡ measured tower ambient temperature

The tower standard day temperature can be obtained using equation 5.83 based on the measured tower pressure altitude.

5.9.1.2.2 Test Aircraft Instrument Corrections

With the test aircraft pressure altitude calculated based on tower measurements, let us now calculate the test aircraft instrument corrected values of altitude, airspeed, total temperature, and Mach number. We can apply the instrument calibrations obtained from the calibration laboratory to calculate the instrument corrected altitude, H_{ic}, airspeed, V_{ic}, and, if required, total temperature, T_{ic}, using equations 5.61, 5.62, and 5.64, respectively. The calculation of instrument corrected Mach number, M_{ic}, can then be performed using H_{ic} and V_{ic} obtained, as follows.

First, for subsonic flight, equation 5.75 can be used to calculate the sea level static port differential pressure ratio, $\frac{q_{c_{ic}}}{P_{a_{SL}}}$. Rearranging this equation yields:

$$\frac{q_{c_{ic}}}{P_{a_{SL}}} = \left[1 + 0.2\left(\frac{V_{ic}}{a_{SL}}\right)^2\right]^{3.5} - 1 \tag{5.97}$$

Based on the value of H_{ic} measured, the static port pressure ratio, δ_s, can be calculated from equation 5.72. This ratio can then be used to calculate the static port differential pressure ratio, $\frac{q_{c_{ic}}}{P_s}$, using the following expression:

$$\frac{q_{c_{ic}}}{P_s} = \frac{q_{c_{ic}}}{P_{a_{SL}}}\frac{1}{\delta_s} \tag{5.98}$$

Finally, with the static port differential pressure ratio calculated, M_{ic} can now be found using equation 5.74.

5.9.1.2.3 Test Aircraft Static Position Error

From equation 5.67, the altitude position error correction, ΔH_{pc}, can be determined by simply taking the difference between the test aircraft pressure altitude, H_c, obtained from the tower measurements and the instrument corrected altitude, H_{ic}. Thus,

$$\Delta H_{pc} = H_c - H_{ic} \tag{5.99}$$

Assuming the total pressure port error is zero, we can use the altitude position error

correction to calculate the remaining desired position error variables. First from equation 5.78, the static port position error ratio, $\frac{\Delta P_p}{P_s}$, is given by

$$\frac{\Delta P_p}{P_s} = \frac{3.61371 \times 10^{-5}}{\theta_s} \Delta H_{pc} \tag{5.100}$$

where the static port temperature ratio, θ_s, can be found from equation 5.83. Now, the static position error pressure coefficient, $\frac{\Delta P_p}{q_{c_{ic}}}$, can be determined from $\frac{\Delta P_p}{P_s}$ and $\frac{q_{c_{ic}}}{P_s}$, as follows:

$$\frac{\Delta P_p}{q_{c_{ic}}} = \frac{\Delta P_p}{P_s} \frac{P_s}{q_{c_{ic}}} \tag{5.101}$$

Additionally, with $\frac{\Delta P_p}{P_s}$, V_{ic}, and M_{ic} known, the airspeed position error correction, ΔV_{pc}, and the Mach position error correction, ΔM_{pc}, can be calculated using equations 5.79 and 5.80 respectively.

5.9.1.2.4 Test Aircraft $\frac{nW}{\delta_s}$ Effects

The significance of angle of attack, or $\frac{nW}{\delta_s}$, effects on our test data must be determined. This is done by correlating the final position error data to each value of this parameter tested. Since test points for the tower flyby method are flown in stabilized 1 g flight ($n = 1$), only the aircraft weight, calculated based on the measured fuel quantity, and the static port pressure ratio determined previously are needed for the calculation.

5.9.1.2.5 Test Aircraft Temperature Probe Recovery Factor

If the test aircraft is equipped with a total temperature probe, the probe's recovery factor should be determined graphically, as discussed in section 5.7.4. The temperature parameter, $\left(\frac{T_{ic}}{T_a} - 1\right)$, is calculated for each test point, where T_a is the ambient temperature measured by at the ground facility and T_{ic} is the instrument corrected total temperature recorded by the test aircraft. The Mach parameter, $\frac{M^2}{5}$, is calculated using the test aircraft Mach number found from equation 5.69, which simply adds M_{ic} and ΔM_{pc} determined previously.

5.9.1.2.6 Data Standardization

Since the test aircraft can never be flown precisely at the same altitude on every pass, the data for each test point are collected at slightly different altitudes. Thus, to properly correlate the position error results, the altitude position error corrections must be standardized to one altitude or recalculated at the standard altitude and weight from the resulting position error pressure coefficient curve for the test aircraft. For the Edwards AFB fly-by line, the standard altitude, $H_{c_{SA}}$, is defined as 2,300 ft PA. To standardize the altitude position error data at the standard altitude, first recall from section 5.7.3.7 the derivation of equation 5.90 which allows us to extrapolate our data to other altitudes at the same values of M_{ic} and ΔM_{pc}. Since we are only standardizing the data by a small increment of altitude (< 200 ft), the assumptions involved in the derivation are valid. Thus, rewriting equation 5.90 we find the new altitude position error is given by:

$$\Delta H_{pc_{SA}} = \Delta H_{pc} \frac{\theta_{SD_{SA}}}{\theta_{SD}} \tag{5.102}$$

where $\theta_{SD_{SA}}$ and θ_{SD} are the standard day temperature ratios corresponding to $H_{c_{SA}}$ and H_c, respectively, calculated from equation 5.18.

While the extrapolation assumed the same value of M_{ic}, the new values of H_{ic} and V_{ic} at the standard altitude must be calculated. The standard altitude instrument corrected altitude can be calculated using equation 5.67, as follows:

$$H_{ic_{SA}} = H_{c_{SA}} - \Delta H_{pc_{SA}} \tag{5.103}$$

Calculation of the standard altitude instrument corrected airspeed first requires the recalculation of the sea level static port differential pressure ratio. Since M_{ic} is the same, the static port differential pressure ratio is also constant for the extrapolation. Thus,

$$\left(\frac{q_{c_{ic}}}{P_{a_{SL}}}\right)_{SA} = \frac{q_{c_{ic}}}{P_s} \delta_{s_{SA}} \tag{5.105}$$

where the standard altitude pressure ratio, $\delta_{s_{SA}}$, is calculated from equation 5.72 at $H_{ic_{SA}}$. Now, we can determine $V_{ic_{SA}}$ from equation 5.75.

Based on our assumptions, the static port position error ratio, $\frac{\Delta P_p}{P_s}$, calculated in equation 5.100, is also a constant for the extrapolation. Therefore, we now know the values all of the parameters required to determine the standard altitude velocity position error, $\Delta V_{pc_{SA}}$, from equation 5.79.

5.9.2 AIRSPEED COMPARISON USING GROUND SPEED COURSE FLIGHT TEST TECHNIQUE

The airspeed comparison using ground speed course (ground speed course) flight test technique provides a method of determining the true airspeed, V_T, of a test aircraft through time and distance measurements. (Reference 7, Section IV.1) It is most commonly used in calibrating air data systems of slow speed aircraft and is extensively used to calibrate helicopter air data systems. The test aircraft is flown at a uniform speed at constant altitude along a designated speed course of known length, as illustrated in Figure 23. The altitude should be as low as safely allowed so that atmospheric properties recorded by ground test engineers on each pass are

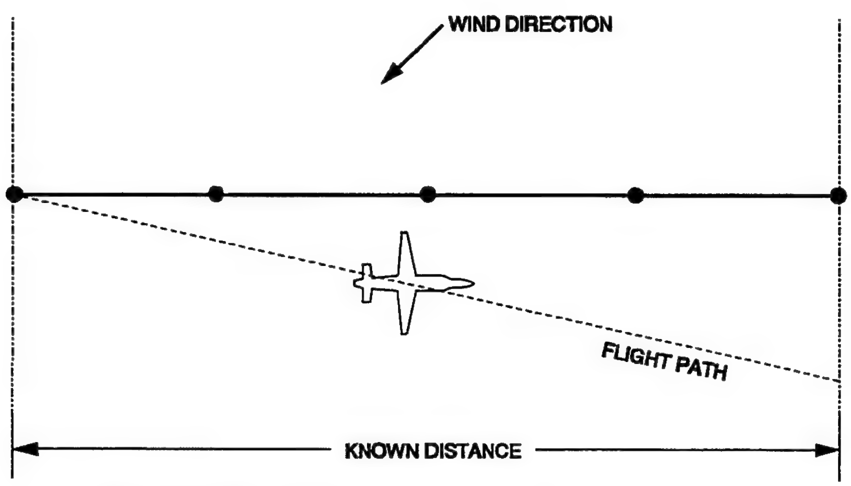


Figure 5.23 Ground Speed Course Diagram

representative of the air mass surrounding the test aircraft. However, the aircraft should be flown at least one wing span above the ground to keep the aircraft out of ground effect. Each test point is repeated on a reciprocal heading so that the effect of wind can be removed by averaging the two ground speeds measured. The resulting average ground speed is assumed to be aircraft true airspeed throughout the air mass. Obviously, this assumption would not be valid on a gusty day or when the winds are frequently varying. Note that no attempt should be made by the pilot to correct the aircraft drift due to crosswind. Data to be recorded in the test aircraft include: altitude, airspeed, fuel, time of day, and if available, total temperature. Depending on the instrumentation available, the time for each pass can be recorded in the cockpit using a stop watch or by ground test engineers using more sophisticated timing devices. Ambient temperature at the midpoint of the speed course should be recorded on each pass by ground test engineers using calibrated instrumentation.

The accuracy of the ground speed course method varies depending on the method of recording the time interval and the wind variation during testing. This accuracy decreases with airspeed; therefore at high speeds, the measurement device must be accurate enough to prevent the error in airspeed to be obscured by errors in the measurement of ground speed. The best time to conduct testing is early in the morning when calm wind and nonturbulent conditions generally prevail. As was true for the tower flyby method, angle of attack, or $\frac{nW}{\delta}$, effects are most prevalent at low speeds; thus, all low speed points should be flown at a variety of gross weights to determine position error corrections over the range of aircraft weight, if possible.

At Edwards AFB, the low altitude speed course is located on Rogers Dry Lake and is approximately 4 miles long. Note that this speed course is marked in statute miles, not nautical miles. Another ground speed course, usually used by helicopters, is located at South Base.

5.9.2.2 DATA ANALYSIS

5.9.2.2.1 Test Aircraft True Airspeed and Mach Number

The test aircraft average ground speed can be calculated by simply dividing the total length of the speed course, L, by the time interval, Δt, measured. Thus,

$$V_{g_{avg}} = \frac{L}{\Delta t} \tag{5.105}$$

Once the values of $V_{g_{avg}}$ have been calculated for a reciprocal heading pair at a desired airspeed, they can be averaged to calculate the test aircraft true airspeed, as follows:

$$V_t = \frac{V_{g_{avg_1}} + V_{g_{avg_2}}}{2} \tag{5.106}$$

To calculate the test aircraft Mach number using V_t, the speed of sound, a, must be determined. We can calculate the speed of sound from equation 5.35 using the average of the ambient temperatures recorded. Although the temperatures were measured by ground test engineers below the test aircraft, it should be representative of the ambient temperature of the air mass surrounding the test aircraft. Thus, the test aircraft Mach number, M, can be found from:

$$M = \frac{V_t}{a} \tag{5.107}$$

5.9.2.2.2 Test Aircraft Instrument Corrections

We can apply the instrument calibrations obtained from the calibration laboratory to calculate the instrument corrected altitude, H_{ic}, airspeed, V_{ic}, and, if required, total temperature, T_{ic}, for each pass using equations 5.61, 5.62, and 5.64, respectively. The calculation of instrument corrected Mach number, M_{ic}, can then be performed using H_{ic} and V_{ic} as discussed previously in section 5.9.1.2.2. Once the instrument corrected parameters have been determined, they must be averaged for each pair of data, before proceeding to the position error or recovery factor analysis.

5.9.2.2.3 Test Aircraft Static Position Error

From equation 5.69, the Mach position error, ΔM_{pc} can be calculated using the previously calculated average values of M and M_{ic}. Thus,

$$\Delta M_{pc} = M - M_{ic} \tag{5.108}$$

Assuming the total pressure port error is zero, we can use ΔM_{pc} to calculate the remaining desired position error variables. First from equation 5.80, the static port position error ratio, $\frac{\Delta P_p}{P_s}$, is given by

$$\frac{\Delta P_p}{P_s} = \frac{1.4\, M_{ic}}{1 + 0.2\, M_{ic}^2}\, \Delta M_{pc} \tag{5.109}$$

We can now calculate the altitude position error correction, ΔH_{pc}, and the airspeed position error correction, ΔV_{pc}, using equations 5.78 and 5.79, respectively, with the aid of equations 5.72 and 5.83 to calculate the static port temperature and pressure ratios. The static position error pressure coefficient, $\frac{\Delta P_p}{q_{c_{ic}}}$, can be determined from $\frac{\Delta P_p}{P_s}$ and $\frac{q_{c_{ic}}}{P_s}$, using equation 5.101. Test data should be correlated to curves of constant angle of attack, or $\frac{nW}{\delta_s}$, as discussed in section 5.9.1.2.4, and standardized as discussed in Section 5.9.1.2.6. These results of this test method can be compared to the those obtained during tower flyby testing to validate the assumption of no total pressure error.

5.9.2.2.4 Test Aircraft Temperature Probe Recovery Factor

If the test aircraft is equipped with a total temperature probe, the probe's recovery factor can be determined, as discussed previously in section 5.9.1.2.5. However, the average values of T_a, T_{ic}, and M for each pair of data must be used in the analysis.

5.9.3 PACER FLIGHT TEST TECHNIQUE

5.9.3.1 TEST METHOD

The pacer flight test technique provides calibration of both altitude and airspeed by direct comparison with another aircraft whose instrument and position error calibrations are known, called the pace aircraft. (Reference 3, Section 5.6.3) The two aircraft are flown level with, and at least half a wing span abreast of, each other to prevent aircraft pressure field interaction. Either aircraft may act as lead. When both aircraft are stabilized at the desired altitude and airspeed, the measured airspeed, altitude, fuel quantity, and total temperature, if available, are simultaneously recorded in each aircraft.

This method of calibration takes less flight time than the other calibration techniques presented, and can cover any *subsonic* airspeed and altitude, as long as the two aircraft are compatible. Angle of attack effects may be eliminated by flying a range of airspeeds at a constant $\frac{nW}{\delta_{ic}}$. However, the angle of attack effects due to weight changes may be small enough that calibrations could be done at a constant altitude rather than a constant $\frac{nW}{\delta_{ic}}$. Therefore, this test method is generally performed at a desired altitude at various airspeeds throughout the test aircraft envelope.

Once corrections have been applied, the pace aircraft provides the pressure altitude, H_c, and calibrated airspeed, V_c, for both aircraft at each test point. This will allow direct calculation of ΔH_{pc} and ΔV_{pc} for the test aircraft. Ambient temperature, T_a, obtained from the pace aircraft can be used to determine the test aircraft temperature probe recovery factor, if required. If the pace aircraft is not equipped with a total temperature probe, the ambient temperature may be obtained from weather balloon measurements, with some loss in accuracy.

5.9.3.2 DATA ANALYSIS

5.9.3.2.1 Pace Aircraft

The pace aircraft air data measurements must first be corrected before being used to calculate the test aircraft corrections. Instrument corrections must be applied to calculate $H_{ic_{PACE}}$, $V_{ic_{PACE}}$, and $T_{ic_{PACE}}$, if required, using equations 5.61, 5.62, and 5.64, respectively. The calculation of $M_{ic_{PACE}}$ can then be performed using $H_{ic_{PACE}}$ and $V_{ic_{PACE}}$ obtained, as was discussed previously in Section 5.9.1.2.2. However, if the test

altitude is above 36,089 ft, equation 5.73 must be used instead of equation 5.72 in the calculation of the static port pressure ratio.

The application of the pace aircraft position error data will be dependent on how the data are provided. If in graphical form, we simply find the pace aircraft position error corrections corresponding to the instrument corrected data. If the data are provided in the form of equation 5.86,

$$\frac{\Delta P_p}{q_{c_{ic}}}\bigg|_{PACE} = f\left(M_{ic_{PACE}},\ \frac{nW}{\delta_s}\bigg|_{PACE}\right)$$

assuming the pace total pressure error was negligible, the position error corrections will have to be calculated. Thus, knowing the pace aircraft weight, calculated from the measured fuel quantity, and the pace static port pressure ratio, $\delta_{s_{PACE}}$, the parameter $\frac{nW}{\delta_s}\big|_{PACE}$ can be calculated. Then the pace static position error pressure ratio, $\frac{\Delta P_p}{q_{c_{ic}}}\big|_{PACE}$, can be determined from the given correction data. Combining this ratio with the pace static port differential pressure ratio, $\frac{q_{c_{ic}}}{P_s}\big|_{PACE}$, obtained from equation 5.98 for the calculation of $M_{ic_{PACE}}$ previously, the pace static port position error ratio can be calculated as follows:

$$\frac{\Delta P_p}{P_s}\bigg|_{PACE} = \frac{\Delta P_p}{q_{c_{ic}}}\bigg|_{PACE}\ \frac{q_{c_{ic}}}{P_s}\bigg|_{PACE} \tag{5.110}$$

The pace position error corrections, $\Delta H_{pc_{PACE}}$ and $\Delta V_{pc_{PACE}}$ can then be calculated using equations 5.78 and 5.79 respectively, with the aid of equation 5.83 or 5.84, as appropriate, for the calculation of $\theta_{s_{PACE}}$. Finally, equations 5.67 and 5.68 can be used to determine the desired values of H_c and V_c.

If required, the pace aircraft measured ambient temperature can be calculated from equation 5.92, given the probe recovery factor, as follows:

$$T_a = \frac{T_{ic_{PACE}}}{1 + \frac{K_{t_{PACE}}}{5} M_{PACE}^2} \tag{5.111}$$

where M_{PACE} is calculated from equation 5.69, with the aid of equation 5.80 for the calculation of $\Delta M_{pc_{PACE}}$.

5.9.3.2.2 Test Aircraft

The test aircraft instrument corrected values of measured air data can be determined as discussed previously in Section 5.9.1.2.2. However, if the test altitude is above 36,089 ft, equation 5.73 must be used instead of equation 5.72 in the calculation of the static port pressure ratio.

With the pressure altitude obtained from the pace aircraft, we can now perform the analysis of Section 5.9.1.2.3 to calculate all of the required test aircraft position error data. However, this analysis assumed that the total pressure error was negligible, allowing the static port position error pressure ratio and the airspeed and Mach position error corrections to be calculated from the altitude position error correction. We can validate this assumption for the test aircraft pitot-static system by using the calibrated airspeed, V_c, obtained from the pace aircraft. From equation 5.68, the airspeed position error correction can be recalculated as follows:

$$\Delta V_{pc} = V_c - V_{ic} \tag{5.112}$$

Comparing this airspeed correction to that obtained from the Section 5.9.1.2.3 analysis will validate or negate our assumption of negligible total pressure error. If the corrections match, the assumption is valid and the position error data can be standardized as discussed in Section 5.9.1.2.6. Again, if the test altitude is above 36,089 ft, equation 5.21 must be used in the calculation of the standard day temperature ratios, and equation 5.73 in the calculation of the standard altitude static port pressure ratio. If the airspeed corrections do not match, the position error analysis of Section 5.9.1.2.3 is invalid and the data cannot be standardized using the equations presented. Only the raw altitude and airspeed position error corrections obtained from equations 5.99 and 5.112, respectively, are valid for final presentation. Regardless of what data are presented in the final form, it still must be correlated to curves of constant angle of attack, or $\frac{nW}{\delta_s}$, as discussed in Section 5.9.1.2.4.

If the test aircraft is equipped with a total temperature sensor, the total temperature probe recovery factor can be determined as discussed in Section 5.9.1.2.5. Again, if ambient temperature was not provided by the test aircraft, it may be obtained from weather balloon measurements. However, the balloon measurements may not be valid for the entire test range, introducing error into the results.

5.9.4 ALTITUDE COMPARISON USING THE TRAILING CONE AND TRAILING BOMB FLIGHT TEST TECHNIQUES

5.9.4.1 TEST METHOD

The altitude comparison using the trailing cone and trailing bomb (trailing cone/bomb) flight test techniques provide methods to measure static pressure, P_a, well below and aft of the test aircraft. This pressure measurement can be directly compared to the static pressure, P_s, measured by the test aircraft to determine the static position error. Due to the high speed instability problems of both methods, they are primarily used for calibration of slow speed aircraft.

In the trailing cone method, a long length of pitot tubing is extended behind the test aircraft. A light weight cone attached to the end stabilizes the tubing and keeps it taut. A schematic of a trailing cone is shown in Figure 5.24 below. The static ports should be located at least six cone diameters ahead of the cone. Due to the uncertainties involved, the trailing cone has been used as a secondary calibration method. The aircraft's pitot-static instruments are calibrated with the trailing cone in place by tower flyby or pace methods. These results are used to calibrate the cone installation. The cone can be used with good results as a recalibration check of that aircraft's instruments or as a primary calibration source for aircraft of the same model. Note that the trailing cone method is limited to airspeeds below 200 knots in order to maintain it below the test aircraft slipstream and due to inherent instabilities.

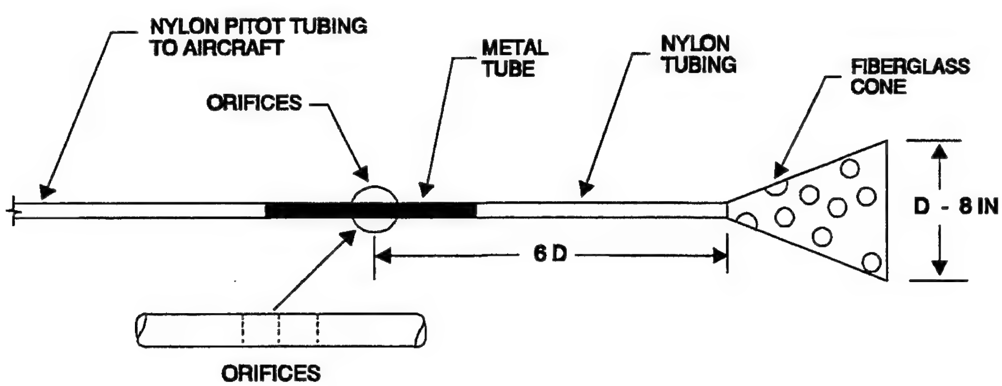


Figure 5.24 Trailing Cone Schematic

The trailing bomb method is similar to the trailing cone method except the end of the pitot tubing is attached to a bomb shaped body with a built in static source, as shown in Figure 5.25. Any error in the bomb's static source is generally determined during wind tunnel testing. At low speeds, the weight of the bomb is enough to keep it below the test aircraft. At higher speeds, the bomb must be fitted with small wings set at a negative angle of attack to keep is out of the slipstream of the aircraft. However, this introduces the instability problems similar to that of a towed glider.

The length of tubing required for either method is at least two wingspans to insure the static pressure source is well outside the test aircraft pressure field. Since this static source is below the aircraft, the static pressure is higher, but the pressure lapse in the tubing is the same as the free stream atmospheric pressure lapse. Thus, if the pitot tubing is attached to an altimeter or pressure gauge next to the aircraft's altimeter, it will indicate free stream pressure at altimeter level.

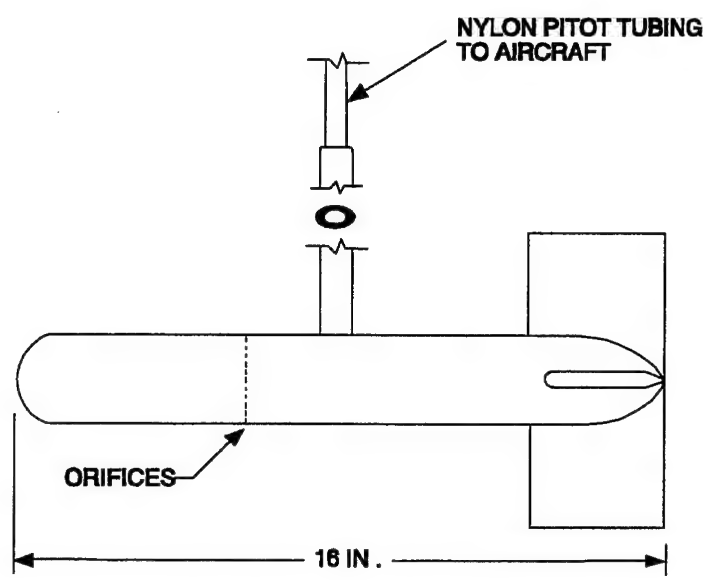


Figure 5.25 Trailing Bomb Diagram

5.9.4.2 DATA ANALYSIS

The test aircraft instrument corrected values are first determined, as discussed in Section 5.9.1.2.2. The static position error calculation is dependent on the particular measurement method selected. If an altimeter were used to measure the pressure provided by the trailing cone or trailing bomb, the altitude measured must first be

corrected for any instrument error to determine the actual test aircraft pressure altitude, H_c. Then the test aircraft static position error corrections can be calculated as discussed in Section 5.9.1.2.3. If a differential pressure gauge were used to measure the static position error directly, or the difference between the static pressure measured by the aircraft and the ambient pressure measured by the trailing bomb or cone, the reading would have to be first corrected for instrument error to determine the actual static position error, ΔP_p. Then the static port position error ratio could be calculated directly using the static port pressure ratio calculated for the test aircraft as follows, instead of from equation 5.100:

$$\frac{\Delta P_p}{P_s} = \frac{\Delta P_p}{\delta_s \, P_{a_{SL}}} \tag{5.113}$$

Once the test aircraft static position error corrections are determined, they can correlated based on $\frac{nW}{\delta_s}$ effects, as discussed in Section 5.9.1.2.4, and standardized as discussed in Section 5.9.1.2.6.

5.9.5 ALTITUDE COMPARISON USING PRESSURE/TEMPERATURE SURVEY FLIGHT TEST TECHNIQUE

5.9.5.1 TEST METHOD

The altitude comparison using pressure/temperature survey (survey) flight test technique is used for calibrations with airspeeds ranging from subsonic to supersonic, supplementing the previous techniques presented. (Reference 7, Section IV-4) This method requires a survey aircraft which can provide accurate pressure altitude and temperature measurements at the test altitude and an accurate means of measuring the inertial positions of the test and survey aircraft. This may be done using ground radar tracking or on-board inertial navigation systems (INS) or global positioning systems (GPS).

An important aspect of this method is that a pressure and temperature survey is required before the test aircraft calibration can be done. The purpose of this survey is to provide a map of pressure altitude and ambient temperature as a function of geometric altitude, North/South position, and East/West position along the route to be flown by the test aircraft. The survey aircraft conducts this survey by flying at a constant airspeed at the test altitude through the air mass along the designated test route. The airspeed selected should be based on the survey aircraft altitude position error correction data available for the test altitude. The survey aircraft inertial position must be recorded as a function of time from the start to finish of the survey.

The survey aircraft crew record time, altitude, airspeed, total temperature, and fuel either during the entire survey or at various points along the survey route, depending on instrumentation.

As soon as possible after completion of the airspace survey, the test aircraft is flown through the surveyed air mass to conduct calibration testing. Test points may include stabilized transonic and supersonic test points or a level acceleration from subsonic to maximum supersonic airspeed followed by a deceleration back to the original subsonic airspeed, depending on the instrumentation installed. However, if a level acceleration/deceleration is flown, the acceleration should be low, since the air data measurements will lag during rapidly changing airspeeds, biasing the data collected. Averaging the results of the acceleration and deceleration will help remove these biases, as long as the velocity changes were fairly consistent. As was done with the survey aircraft, inertial position data is taken during testing. Data recorded by the test aircraft include: time, altitude, airspeed, fuel, and, if available, total temperature.

If radar tracking is used to measure inertial data, the test and survey aircraft must have a tracking beacon for accurate radar ranging and to allow ground controllers to provide course corrections when necessary. Obviously, an accurate time correlation is necessary to properly inertial position data to aircraft recorded air data and weight. The test aircraft pressure altitude can be found by simply correlating the test aircraft geometric altitude with the survey results for the particular position in space. By adding the difference between the survey and test geometric altitudes, corrected for non-standard temperature, to the survey pressure altitude, the test aircraft pressure altitude can be determined. Note that the geometric altitude from the Flight Test Center tracking radar is accurate to approximately 25 feet, while precision coded GPS with differential corrections is accurate to approximately ten feet. The basic assumption of this method is that the pressure and temperature of the surveyed airspace remain constant during testing.

5.9.5.2 DATA ANALYSIS

5.9.5.2.1 Pressure and Temperature Survey

The calculation of the survey aircraft pressure altitude, $H_{c_{Surv}}$, and ambient temperature, $T_{a_{Surv}}$, can be performed using measured air data as was presented for the pace aircraft in Section 5.9.3.2.1. These data can then be time correlated with the geometric altitude, h_{Surv}, North/South position, X_{Surv}, and East/West position, Y_{Surv} measured during the survey run.

5.9.5.2.2 Test Aircraft Pressure Altitude

To calculate the test aircraft pressure altitude, H_c, the test aircraft measured air data must first be time correlated with the geometric altitude, h, North/South position, X, and East/West position, Y. These test aircraft data are then matched with the pressure survey results based on North/South and East/West positions. Once all data has been properly correlated, H_c can be calculated from the following equation for each test point:

$$H_c = H_{c_{Surv}} + (h - h_{Surv}) \frac{T_{a_{SD}}}{T_{a_{Surv}}} \tag{5.114}$$

where equation 5.28 has been incorporated to convert the geometric altitude difference to a pressure altitude difference.

5.9.5.2.3 Test Aircraft Instrument and Static Position Error Corrections

The test aircraft instrument corrected values of measured air data can be determined as discussed previously in Section 5.9.1.2.2. However, if the test altitude is above 36,089 ft, equation 5.73 must be used instead of equation 5.72 in the calculation of the static port pressure ratio. Additionally, if supersonic, equation 5.77 must be used to calculate the sea level static port differential pressure ratio, and equation 5.76 must be solved iteratively to calculate the instrument corrected Mach number.

With the pressure altitude calculated, we can now perform the analysis of Section 5.9.1.2.3 to calculate all of the required test aircraft position error data. Again, this analysis assumed that the total pressure error was negligible, allowing the static port position error pressure ratio and the airspeed and Mach position error corrections to be calculated from the altitude position error correction. Note that if the test point is supersonic, equations 5.81 and 5.82 must be used to calculate the airspeed and Mach number position error corrections. Once calculated, the position error data can be standardized as discussed in Section 5.9.1.2.6. If the test altitude is above 36,089 ft, equation 5.21 must be used in the calculation of the standard day temperature ratios, and equation 5.73 in the calculation of the standard altitude static port pressure ratio. In addition, if the test point is supersonic, equation 5.77 must be solved iteratively to calculate the standard altitude instrument corrected airspeed. Final presentation of position error correction data must again be correlated to curves of constant angle of attack, or $\frac{nW}{\delta_s}$, as discussed in Section 5.9.1.2.4.

If the test aircraft is equipped with a total temperature probe, the probe's recovery factor can be determined, as discussed previously in Section 5.9.1.2.5.

5.9.6 ALTITUDE COMPARISON USING SMOKE-LAYING AIRCRAFT FLIGHT TEST TECHNIQUE

5.9.6.1 TEST METHOD

Similar to the altitude comparison using pressure/temperature survey method, the altitude comparison using smoke-laying aircraft (smoke trail) flight test technique provides altitude calibration of the test aircraft for airspeeds from subsonic to supersonic. (Reference 7, Section IV-4) This technique requires a smoke-laying aircraft calibrated for at least one airspeed at the test altitude(s). The smoke trail allows the test aircraft to precisely fly the same path through the air mass, eliminating the need for inertial measuring devices to correlate data.

The smoke-laying aircraft is first flown at constant airspeed at the test altitude along the designated ground track, providing a constant *pressure* altitude smoke trail, as illustrated in Figure 5.26. The airspeed is chosen where the position error is well defined for the test altitude. Data recorded by the crew of the smoke laying aircraft include fuel, deviations from the target altitude and airspeed, and if required, total temperature. Immediately after completion of the smoke trail, the test aircraft performs a series of stabilized test points in the high subsonic to supersonic airspeed range, or a level acceleration from subsonic to maximum supersonic followed by a deceleration to the original subsonic airspeed, along the smoke trail. Data recorded by the test aircraft includes altitude, airspeed, and, if available, total temperature. Obviously, the accuracy of this test is dependent on how precisely the smoke-laying aircraft maintains altitude constant and how precisely the test aircraft is flown along the smoke trail. In addition, the smoke generating device must be located outside the influence of the aircraft downwash and the test can only be conducted when the winds are calm at the test altitude. Otherwise, the smoke trail will not provide an accurate path through the air mass for the test aircraft.

5.9.6.2 DATA ANALYSIS

The calculation of the pressure altitude of the smoke trial, and if required, the ambient temperature of the surrounding air mass can be performed using air data measured by the smoke-laying aircraft, as presented previously for the pace aircraft in Section 5.9.3.2.1. This altitude is assumed constant for all test points flown by the test aircraft. The calculation of the test aircraft instrument and position error corrections can be performed as presented in Section 5.9.5.2.3 for the radar method. Since there is no position correlation along the smoke trail, variation in ambient

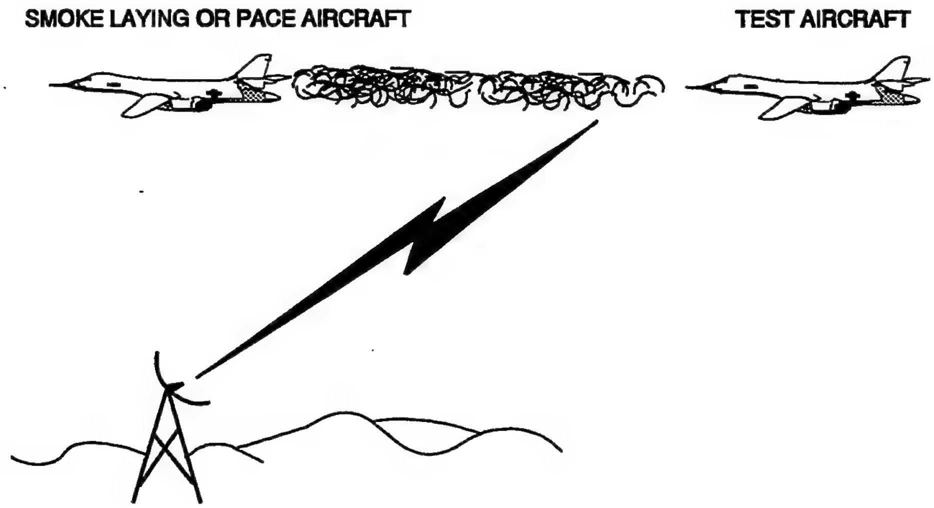


Figure 5.26 Radar/Smoke Trail Method

temperature along the test route may introduce error in the calculation of the test aircraft temperature probe recovery factor, if this analysis is required.

5.9.7 AIRSPEED COMPARISON USING ALL ALTITUDE SPEED COURSE FLIGHT TEST TECHNIQUE

5.9.7.1 TEST METHOD

The airspeed comparison using all altitude speed course (all altitude speed course) flight test technique provides a method of determining the true airspeed, V_T, of a test aircraft at any altitude and airspeed based on ground radar, inertial navigation unit (INU), or global positioning system (GPS) ground speed measurements. (Reference 3, Section 5.6.5) This method requires ambient temperature measurement from either a calibrated total temperature probe on the test aircraft or from a weather balloon, and, an estimate of the wind direction for each test point.

While there are a variety of test procedures which allow the elimination of wind from ground speed, an effective procedure developed by Maj Norman Howell was recently tested by Test Pilot School students (Reference 8). The test aircraft is stabilized at the test airspeed and altitude in a direction perpendicular to the estimated wind direction. Ground speed and horizontal flight path angle for each test point are recorded from ground radar, INU, or GPS measurements. Time, indicated airspeed, indicated altitude, total temperature, and true heading angle are recorded in the test

aircraft. The test point is then repeated on a reciprocal heading. The validity of the test point pair is determined based on a comparison of the flight path deviation angles and true airspeeds calculated. Matches in these parameters validate that the aircraft was (1) actually flown at 90° to the wind and (2) that the wind did not vary between test points. If a match is made, the resulting true airspeeds are averaged. Note that if necessary, ambient temperature from balloon data can be used, with some loss in accuracy. If ground radar is used, the test aircraft should be equipped with a tracking beacon for accurate radar ranging and to allow ground controllers to provide course corrections, when necessary. In addition, accurate time correlation is necessary to properly correlate ground recorded data to aircraft recorded data.

5.9.7.2 DATA ANALYSIS

5.9.7.2.1 Test Aircraft True Airspeed and Mach Number

Independent of the source of measurement, the ground speed can be calculated by simply taking the square root of the sum of the squares of the horizonal inertial velocity components. Therefore,

$$V_g = \sqrt{V_x^2 + V_y^2} \tag{5.115}$$

The flight path deviation angle, $\Delta\sigma$, can be calculated as follows:

$$\Delta\sigma = \sigma - \psi \tag{5.116}$$

Knowing the flight path deviation, and, assuming the aircraft were flown precisely at 90° to the wind, the true airspeed can be found:

$$V_t = V_g \cos(\Delta\sigma) \tag{5.117}$$

Combining equations 5.35, 5.36 and 5.111, we can develop the following new equation to calculate the ambient temperature from the true airspeed:

$$T_a = T_{ic} - \frac{K_t}{5}\frac{V_t^2}{\gamma R} = T_{ic} - \frac{K_t V_t^2}{7590.7 \frac{kt^2}{K}} \tag{5.118}$$

The speed of sound and test aircraft Mach number can then be calculated from equations 5.35 and 5.107, respectively.

5.9.7.2.2 Test Aircraft Instrument and Position Error Corrections

The test aircraft instrument corrected values of measured air data can be determined as discussed previously in Section 5.9.1.2.2. However, if the test altitude is above 36,089 ft, equation 5.73 must be used instead of equation 5.72 in the calculation of the static port pressure ratio. Additionally, if supersonic, equation 5.77 must be used to

calculate the sea level static port differential pressure ratio, and equation 5.76 must be solved iteratively to calculate the instrument corrected Mach number.

With the Mach number known, we can now perform the analysis of Section 5.9.2.2.3 to calculate all of the required test aircraft position error data. Again, this analysis assumed that the total pressure error was negligible, allowing the static port position error pressure ratio and the airspeed and altitude position error corrections to be calculated from the Mach number position error correction. If the test point is supersonic, from equation 5.82, the static port position error ratio is then given by:

$$\frac{\Delta P_p}{P_s} = \frac{7\left(2 M_{ic}^2 - 1\right)}{M_{ic}\left(7 M_{ic}^2 - 1\right)} \Delta M_{pc} \tag{5.119}$$

and the velocity position error correction then calculated using equation 5.81. The position error data can be standardized as discussed in Section 5.9.1.2.6. However, if the test altitude is above 36,089 ft, equation 5.21 must be used in the calculation of the standard day temperature ratios, and equation 5.73 in the calculation of the standard altitude static port pressure ratio. In addition, if the test point is supersonic, equation 5.77 must be solved iteratively to calculate the standard altitude instrument corrected airspeed. Final presentation of position error correction data must again be correlated to curves of constant angle of attack, or $\frac{nW}{\delta_s}$, as discussed in Section 5.9.1.2.4.

5.10 EXTRACT FROM MIL-P-26292C

The following sections are taken from MIL-P-26292C (USAF), Military Specification Pitot and Static Pressure Systems, Installation and Inspection of (Reference 5).

4.5 Test Methods

4.5.1 Examination of Installation. A visual examination of the pitot-static system installation shall be conducted to determine conformance to the requirements of this specification.

4.5.2 Electrical Wiring. Rated voltage shall be supplied by the primary electrical source. The voltage measured across the terminals of the pitot or pitot-static tube shall be not less than 26V for a 28V primary source and shall be not less than 100V for a 115V primary source when heaters are energized.

4.5.3 Pressure Leakage

4.5.3.1 Tubing Pressure Leak. The installed pitot-static tubing shall be individually tested with pneumatic equipment disconnected. The outlets of each tube shall be sealed and a standard altimeter, having a minimum known leak rate, shall be installed on one outlet of the static pressure line. A vacuum sufficient to provide an indication of 50,000 feet on the altimeter shall be applied to the static pressure source (pitot-static tube or flush static port) and then cut off. The leakage, with corrections made for the known indicator leak rate, shall not exceed a 100-foot altitude after a 5-minute period. A pressure equivalent to the total pressure encountered at the maximum speed of the aircraft shall be applied to the total pressure source (pitot-static tube or pitot tube) and then cut off. After a period of 5 minutes, the leakage, with corrections made for the known indicator leak rate, shall not exceed an airspeed of 1.0 knot.

4.5.3.1.1 Alternate Test Procedure for Missiles. The installed pitot-static tubing shall be individually tested with all pneumatic equipment disconnected. The outlets of each tube shall be sealed and an accurate mercury manometer installed on one end of each tube at the position of the major pressure transducer. A vacuum equal to 86.99 mm Hg shall be applied to the static pressure source (pitot-static tube or flush static ports) and then cut off. After a 5-minute period, the pressure leakage shall not exceed an amount equivalent to an airspeed of 1.0 knot.

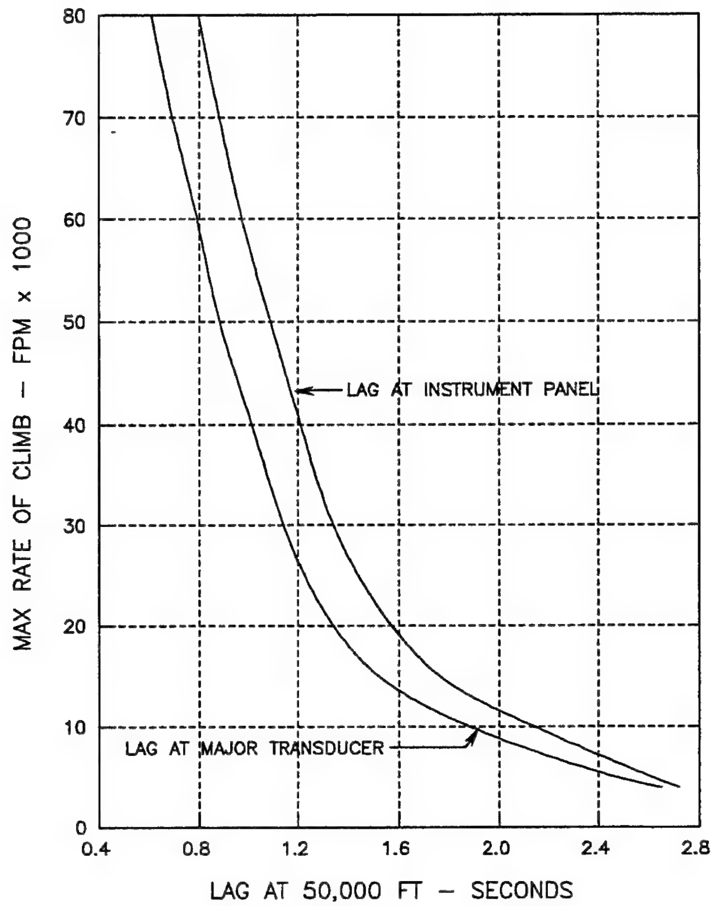
4.5.3.2 Pitot-Static System Leakage. With the instruments and related equipment properly connected to the pitot and static pressure lines, the entire system shall be tested for leakage. A vacuum sufficient to provide an indications of an altitude of 40,000 feet or three-fourths of full-scale deflection on the production altimeter, whichever is less, shall be applied to the static pressure system and then cut off. after a period of 5 minutes, the leakage shall not exceed an altitude drop of 3,000 feet. A pressure equal to that obtained with the aircraft at maximum airspeed shall be applied to the total pressure source and then cut off. After a period of 5 minutes, the leakage indicated on the airspeed indicator shall not exceed 10 knots.

4.5.3.2.1 Alternate Procedure for Missiles. With the instruments and other related pneumatic equipment properly connected to the pitot and static pressure lines, the entire system shall be tested for leakage. The outputs of the major pressure transducer shall be monitored for determination of leakage. A vacuum sufficient to provide a signal equivalent to 40,000 feet shall be applied to the static pressure sources and them cut off. After a period of 5 minutes, the leakage as indicated by the transducer output signal shall not exceed an amount equivalent to 10 knots.

4.5.4 Flushness and Smoothness (Flush Static Port Installation). The flush static plate installation on the aircraft shall be measured with a suitable device and shall not deviate more than 0.0005 inch per inch from the mean shape of the aircraft mold line. The step between the flush and the aircraft shall not exceed 0.010 inch. In the case of compensated flush static plates, deviation from the mean shape of the mold line shall be subject to the approval of the procuring activity.

4.5.5 Pressure Lag. During the design phase, the contractor shall submit to the procuring activity a theoretical lag analysis of the proposed static pressure system. The analysis shall be similar to that explained in the airborne pressure measuring system method (see 6.2), and shall include a schematic of the static pressure system with tubing dimensions and instrument volumes.

4.5.5.1 Verification of Response Characteristics. The contractor shall conduct a test, either on a mockup of the system or on the actual installation on the aircraft, to verify the response characteristics of the production static pressure system. The lag of the system shall be determined at 50,000 feet and shall not exceed the limits shown on Figure 6. (The maximum rate of climb shown in the ordinate of Figure 6 refers to the maximum rate of climb capability of the aircraft). The data shall be submitted to the procuring activity prior to production release of the aircraft.



MIL-P-26292C Figure 6 Static Pressure System Lag

4.5.6 Installation Error. For missile applications, the contractor shall submit to the procuring activity for approval (1) applicable accuracy and response requirements based on the required performance of the vehicle (2) a proposed test program to determine the pitot-static pressure installation error. For manned aircraft, the pitot-static installation error shall be determined by one of the following methods, or by a combination of these methods, or by other methods, upon approval by the procuring activity, capable of producing equal or superior results. Methods used shall be compatible with the performance requirements of the vehicle and as specified herein.

a. Tower flyby method - The aircraft is flown close to an aircraft control tower and the altitude of the aircraft is measured by photographing its position on a measured grid.

b. Radar phototheodolite method - The aircraft is tracked by radar using a boresight camera to correct the azimuth and elevation angles read from the radar scales.

c. Pacer technique - The pacer aircraft and the aircraft being calibrated are flown together in close formation while the altitude and airspeed indications are compared.

d. Flyby parallax technique - The pacer aircraft is flown at a constant speed and altitude while the aircraft to be calibrated alternately decelerates and accelerates while keeping the pacer aircraft in line with the horizon.

e. Trailing cone method - A static port is suspended far enough behind an aircraft so that it is unaffected by the aerodynamic disturbances of the aircraft. The altitude readings are then compared with those readings obtained from the trailing cone.

4.5.6.1 Outline. An outline of the method and instrumentation to be used for airspeed and altimeter system calibration shall be submitted to the procuring activity and approval obtained 60 days prior to the initiation of tests.

4.5.6.2 Position Error Tolerance. In the clean configuration and throughout the full weight range of the aircraft, the static pressure position error shall not exceed the envelope of tolerances specified on Figure 7. If the position error falls within the envelope of curve B but not in the envelope of curve A, compensation through an air data computer is necessary. If the position error is entirely within the envelope of curve A then no air data computer correction is required. The exact tolerance envelope for curve A is as specified in Table I. Throughout the entire supersonic range, the position error ($\Delta P/q_c$) shall not exceed ±1.0 percent for flush static port installation nor ±0.4 percent for nose-boom installations. In the landing configuration below mach [sic] 0.4 with the landing gear and wing flaps down, the altitude tolerance shall be ±30 feet. Operation of speed brakes, fire control doors, gun ports, bomb bay doors, landing gear, flaps, or similar equipment shall not cause the airspeed or altitude calibration to vary by more than ±2 knots or ±15 feet, whichever is the least from the clean configuration. The limits specified are not intended to include instrument indicator errors. The calibration data shall be submitted in a form

substantially as outlined on Figures 7 and 8. The altitude calibration system shall include annotation of angle of attack during the various aircraft maneuvers.

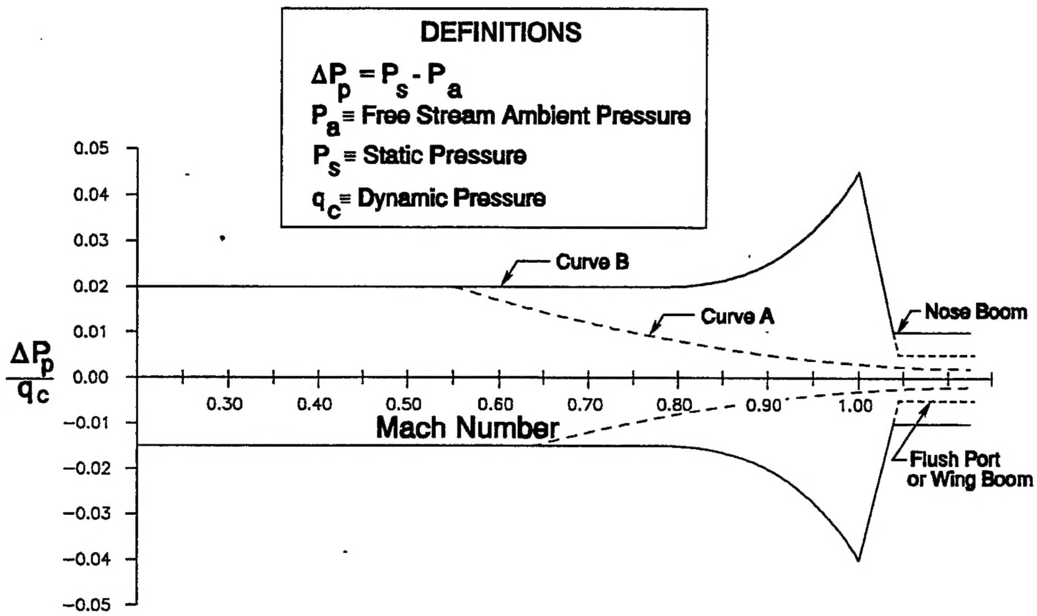
MIL-P-26292C TABLE 1 Curve Tolerance

Mach Number	*Position Error Pressure Coefficient,* $\Delta P / q_c$
0.3	-0.015 to +0.020
0.4	-0.015 to +0.020
0.5	-0.015 to +0.020
0.6	-0.015 to +0.017
0.7	-0.012 to +0.012
0.8	-0.008 to +0.008
0.9	-0.005 to +0.005
1.0	-0.003 to +0.003
1.1	-0.002 to +0.002
1.2	-0.002 to +0.002

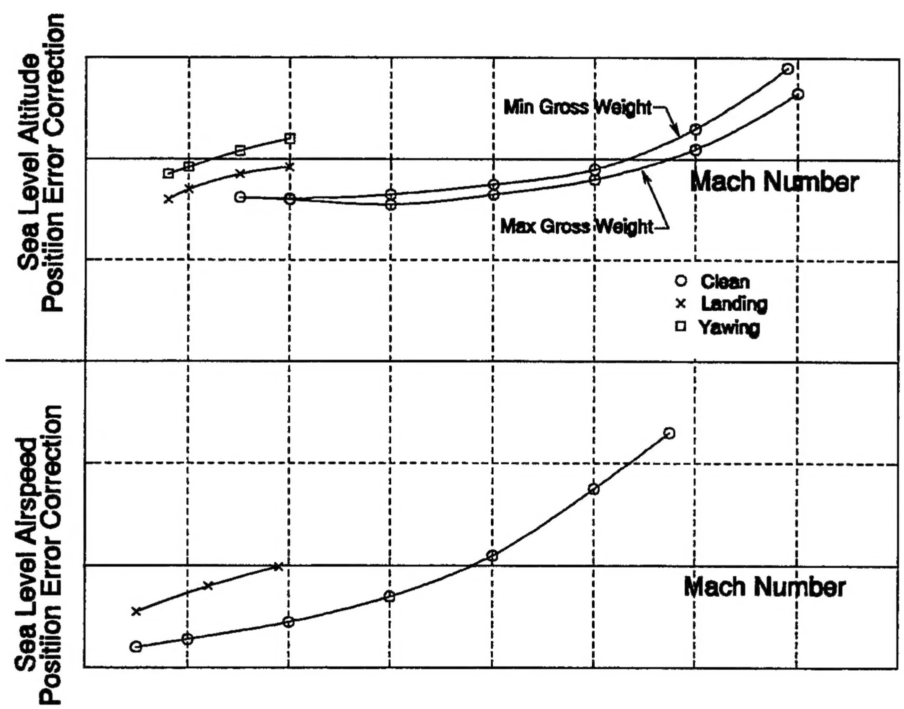
4.5.6.3 The measured pitot pressure shall not differ from the true pitot pressure by more than 0.4 percent throughout the entire mach [sic] number range of the aircraft.

4.5.7 <u>Dynamic Flight Conditions</u>

4.5.7.1 <u>Yawing</u>. While the aircraft is yawed approximately 10° (or full right and left rudder, whichever is less) in both right and left directions during approach conditions, the airspeed indication shall not differ from the straight-flight conditions by more than ±2.0 knots. The altimeter indication shall not differ from the straight-flight conditions by more than a ±20-foot altitude. Effects of yawing conditions shall be demonstrated in the missile test program.



MIL-P-26292C Figure 7 Position Error Tolerance



MIL-P-26292C Figure 8 Data Presentation

4.5.7.2 Pullup. A rate-of-climb indicator shall be connected to the static pressure system (the pilot's and copilot's instruments may be used). Variation of the static pressure during pullups from straight and level flight shall be determined at a safe altitude above the ground and at a minimum of three widely separated indicated airspeeds. During an abrupt pullup from level flight, the rate of climb indicator shall indicate UP without excessive hesitation and shall not indicate DOWN before it indicates UP.

4.5.7.3 Pushover. A rate-of-climb indicator shall be connected to the static pressure system (the pilot's and copilot's instruments may be used). Variation of the static pressure during pushover from straight and level flight shall be determined at a safe altitude above the ground and a minimum of three widely separated indicated airspeeds. During an abrupt pushover from level flight, the rate-of-climb indicator shall indicate DOWN without excessive hesitation and shall not indicate UP before it indicates DOWN.

4.5.7.4 Rough Air. Sufficient maneuvering shall be accomplished in flight to determine installation of the pitot-static system will produce no objectionable instrument pointer oscillation in rough air. Pointer oscillation of the airspeed indicator shall not exceed 3 knots.

6. NOTES

6.1 Intended use. This specification is intended to be used as a basis for the installation and inspection of pitot and static pressure systems in aircraft and missiles.

6.2 Information. Information on flush static pressure port calibrations is contained in SEG Technical Report 65-3 entitled "Evaluation of Factors Affecting the Calibration Accuracy of Aircraft Static Pressure Systems". Information on airborne pressure measuring systems is contained in WADC Technical Report 57-351 entitled "The Influence of Geometry Parameters upon Lag Error in Airborne Pressure Measuring Systems". Contractors having active contracts may obtain these documents from the Defense Documentation Center, Cameron Station, Building 5, 5010 Duke Street, Alexandria, Virginia 22324. Prospective bidders should contact the Aeronautical Systems Division, ASNFI-10, Wright-Patterson Air Force Base, Ohio.

5.11 REFERENCES

1. Anderson, John D. Jr., Introduction to Flight, 2nd Edition, New York: McGraw-Hill Book Company, 1985.

2. U.S. Standard Atmosphere, 1976, National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration, United States Air Force, October 1976

3. Herrington, Shoemacher, Bartlett, and Dunlap, Flight Test Engineering Handbook. FTC-TR-6273, Air Force Flight Test Center, January 1966.

4. Hendrix, Frazier, Godwin, and Durnin, Performance Flight Testing Theory, FTC-TIH-64-2005, Air Force Flight Test Center, December, 1965.

5. MIL-P-26292C (USAF), Military Specification, Pitot-Static Pressure System, Installation and Inspection of, 3 December 1969.

6. Bogue, Rodney K., Recent Flight-Test Results of Optical Airdata Techniques, AIAA-92-4086, 6th AIAA Biennial Flight Test Conference, August 1992.

7. DeAnda, Albert G., AFFTC Standard Airspeed Calibration Procedures, FTC-TIH-68-1001, Air Force Flight Test Center, April 1968.

8. Bailey, William D., Investigation of Using Global Positioning System for Air Data System Calibration of General Aviation Aircraft, AFFTC-TR-95-76, Air Force Flight Test Center, January 1996.

5.12 BIBLIOGRAPHY

1. Flight Test Manual, Vol. 1, Advisory Group for Aeronautical Research and Development North Atlantic Treaty Organization. Oxford: Pergamon Press, 1962.

2. Hendrix, Frazier, Godwin, and Durnin, Performance Flight Testing Theory, FTC-TIH-64-2005, Air Force Flight Test Center, December, 1965.

3. Rosemount Engineering Company, Electrical and Temperature Instruments, Bulletin 7637.